Topological Phases Derived from Discrete Higher Gauge Theory and Representations of the Loop Braid Group

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RPG-2018-029: "Emergent Physics From Lattice Models of Higher Gauge Theory"

(Discrete) gauge theory and holonomy

- Let M be a manifold.
- A path in M is a piecewise smooth map $\gamma\colon [0,1] \to M$.
  We consider paths up to homotopy, relative to the end-points.
- Denote paths as $(x \xrightarrow{\gamma} y)$, x and y are initial and end-points.
- Paths $(x \xrightarrow{\gamma} y)$ and $(y \xrightarrow{\gamma'} z)$ can be concatenated into another path

$$(x \xrightarrow{\gamma} y)(y \xrightarrow{\gamma'} z) = (x \xrightarrow{\gamma\gamma'} z).$$

- Given a subset $M^0 \subset M$, we utilize the groupoid: $\Pi_1(M, M^0)$
- Set of objects is M^0. Morphisms $x \to y$ are paths from x to y.

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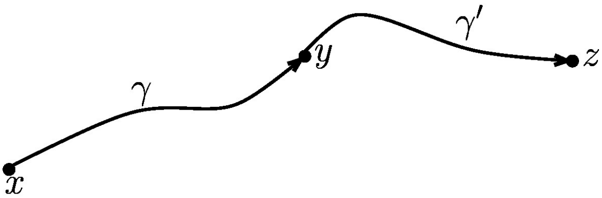

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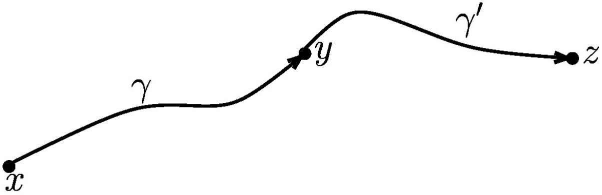

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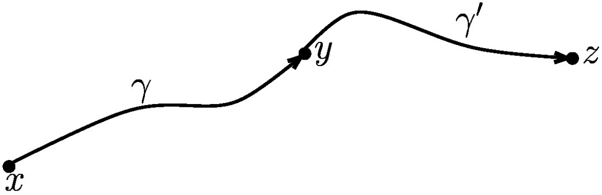

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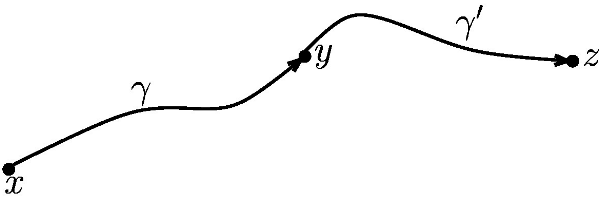

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Let G be a group. G will be finite throughout. M a manifold.
Given a principal G-bundle $P \to M$, and local trivialisations, we have the parallel transport of P.

$$\mathcal{F} : \{Paths \quad in \;\; M\} \to G$$
$$\gamma \longmapsto \mathrm{hol}^1(\gamma) = g_\gamma \in G$$

(Parallel transport is also called "holonomy".)
Recall parallel transport preserves concatenation of paths:

$$\mathcal{F}((x \xrightarrow{\gamma} y)(y \xrightarrow{\gamma'} z)) = \mathcal{F}(x \xrightarrow{\gamma} y)\ \mathcal{F}(y \xrightarrow{\gamma'} z)$$

Let $M^0 \subset M$. Principal G-bundles P hence give rise to functors

$$\mathcal{F} : \Pi_1(M, M^0) \to G,$$

and $\mathcal{F}$ completely determines P.
NB: must specify elements $p_v \in F_v$, the fibre of P at $v \in M^0$.
If G is a Lie group, we need G-connection A in P.
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Gauge Theory and Holonomy

Conversely, G-connections can be defined from their holonomy.
If G is finite, and M compact, we only need to know the holonomy along a finite number of paths. The theory becomes combinatorial.
Combinatorially, a G-connection over M looks like:

where $a, b, c, d, e, f, g \in G$.

There are relations that must be satisfied on triangles.
The holonomy around each should be trivial.

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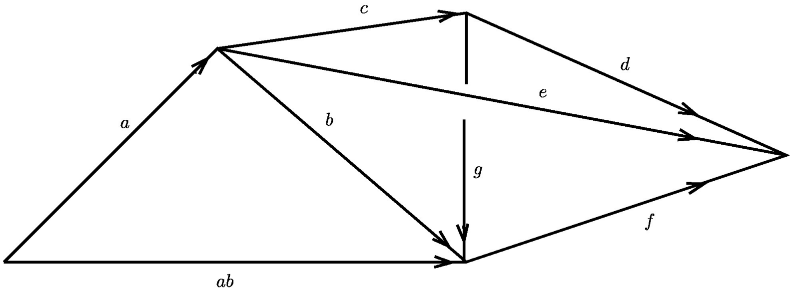

where $a, b, c, d, e, f, g \in G$.

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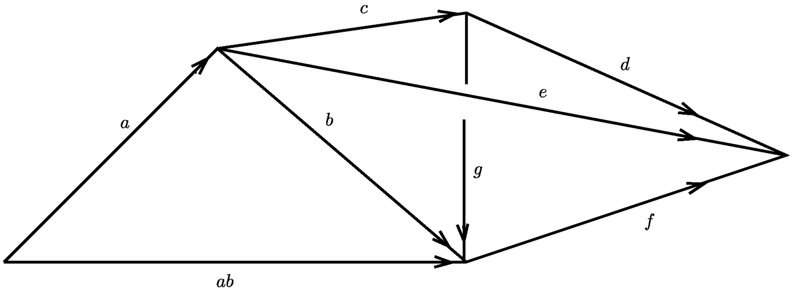

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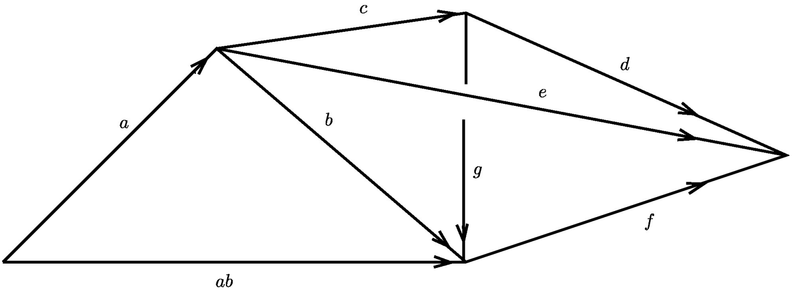

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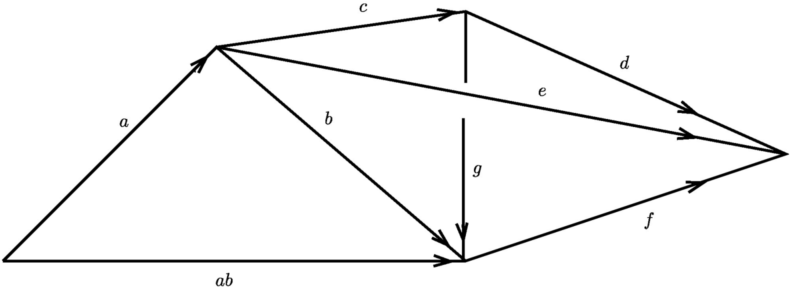

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Kitaev Quantum Double Model

- Let M be a manifold with a decomposition L into 'cells': vertices $v \in L^0$, edges $t \in L^1$, plaquettes $P \in L^2$, blobs $b \in L^3$

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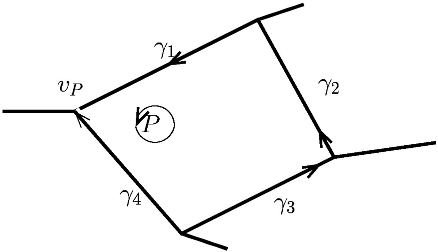

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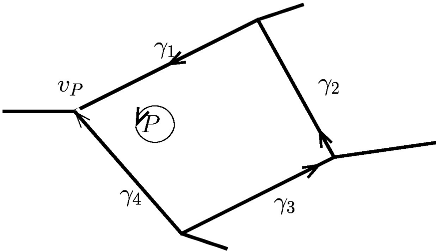

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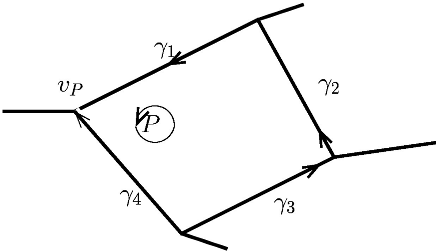

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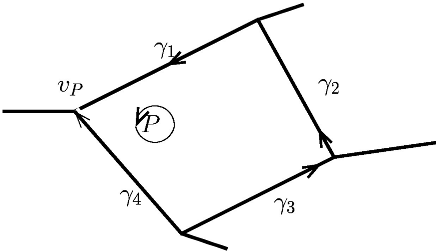

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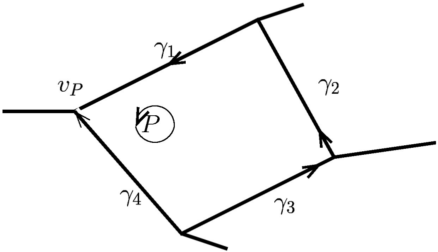

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- $T(M, L) = \prod_{v \in L^0} G = \{Functions\ L^0 \to G\},$
  the group of gauge operators U.
- Given $v \in L^0$, $g \in G$, put $U_v^g \in T(M, L)$ to be the gauge operator (called vertex operator) such that:
  $U_v^g(x) = \begin{cases} g, & \text{if } x = v \\ 1_G, & \text{otherwise} \end{cases}$
- Left-action of $T(M, L)$ on $V(M, L)$, by gauge transformations:
  Let $\mathcal{F} \in \hom(\Pi_1(M^1, M^0), G)$ and $U \in T(M, L)$
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(Slightly different language, as in 1702.00868 [math-ph])
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Theorem: The ground state $GS(M,L)$ of H is:

$$GS(M,L) = \{\mathcal{F} \in \mathbb{C}(\hom(\Pi_1(M, M^0), G) : U \triangleright \mathcal{F} = \mathcal{F}, \forall U \in T(L,G)\}$$

$GS(M,L) \cong \mathbb{C}\{Maps\ M \to B_G\}/homotopy$, canonically.
Here B_G is the classifying space of G.

Hence $GS(M,L) = V(M)$ does not depend on L and only on M.

The Kitaev Quantum Double Model (quant-ph/9707021)

(Slightly different language, as in 1702.00868 [math-ph])
M with CW-decomposition L. $V(M,L) = \mathbb{C}\,\hom(\Pi_1(M^1, M^0), G)$.
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Extension to Higher Gauge Theory

- Higher gauge theory is a higher order version of gauge theory.
- Higher gauge theory allows us to formalise non-abelian holonomy along paths, and also non-abelian holonomy along surfaces.
- Higher order version of a group: a "2-group".
- 2-groups are equivalent to crossed modules.

  A crossed module of groups $\mathcal{G} = (\partial\colon E \to G, \triangleright)$ is given by:
  - a group map $\partial\colon E \to G$,
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Examples of crossed modules

Let G be a group with a left-action $\triangleright$ on an abelian group A, by automorphisms.
Put $\mathcal{G} = (A \xrightarrow{a \in A \mapsto 1_G} G, \triangleright)$.

In the general example above put:

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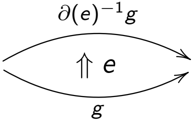

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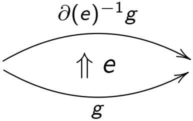

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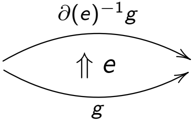

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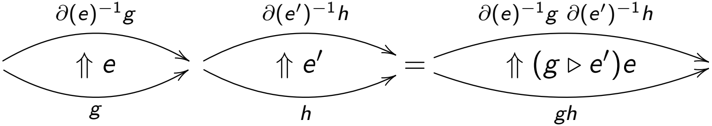

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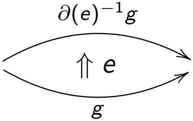

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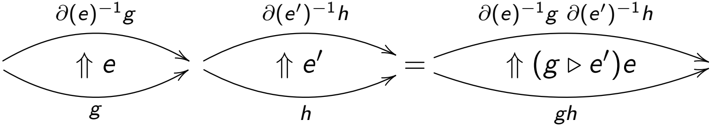

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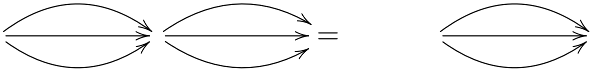

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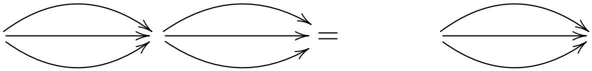

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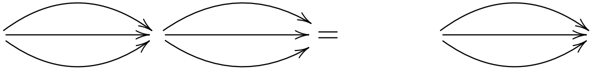

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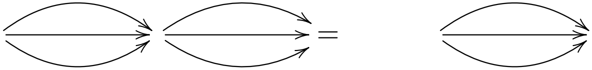

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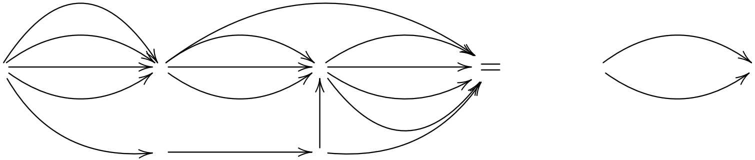

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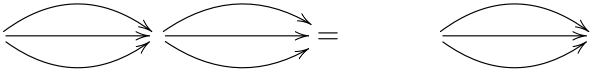

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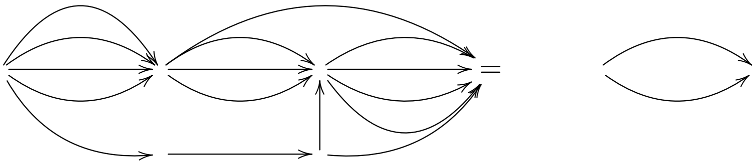

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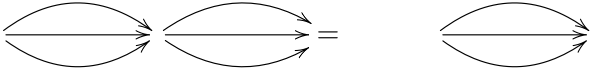

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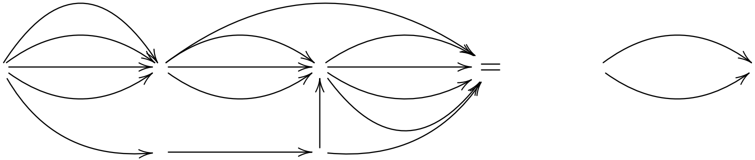

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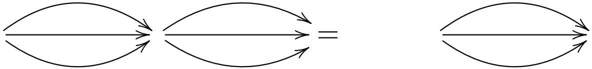

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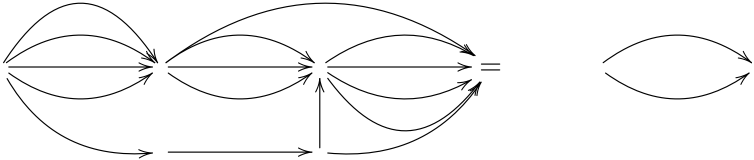

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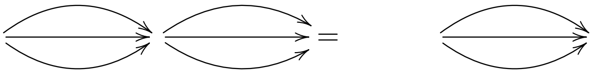

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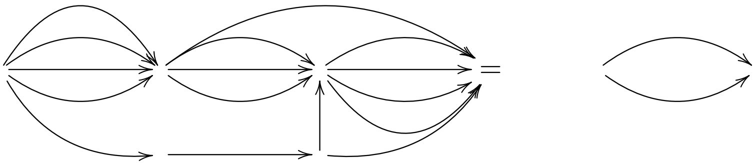

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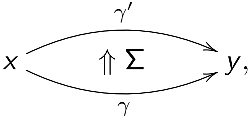

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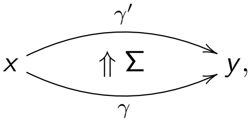

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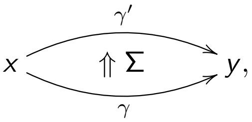

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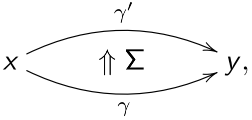

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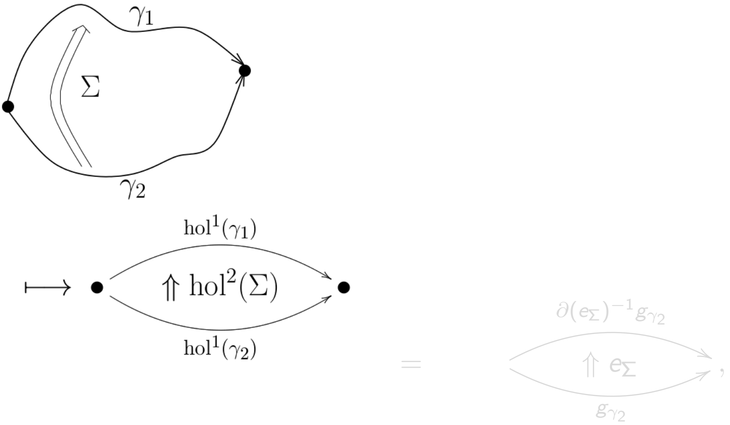

Note: for Lie crossed modules $(\partial\colon E \to G, \triangleright)$, 2-dimensional holonomies arise from pairs $A \in \Omega^1(M, \mathfrak{g})$ and $B \in \Omega^2(M, \mathfrak{e})$, with $\partial(B) = Curv_A = dA + \frac{1}{2}[A, A]$.

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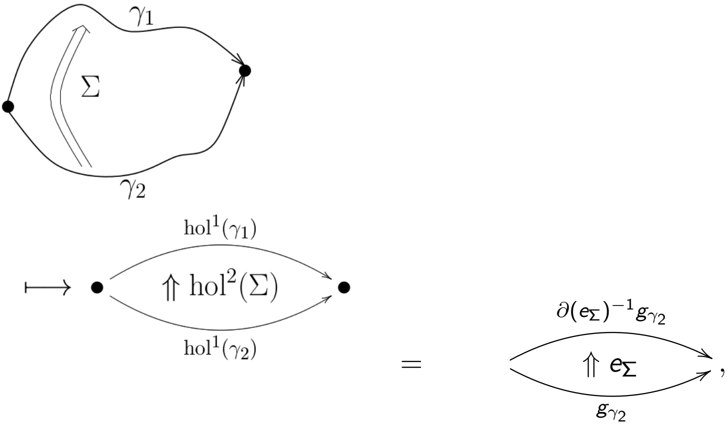

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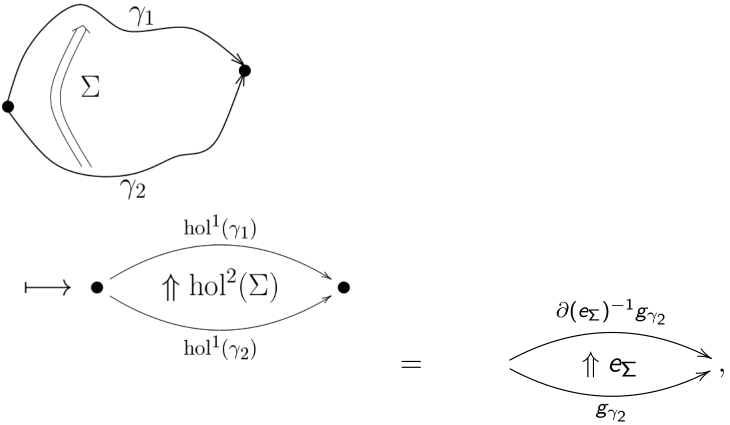

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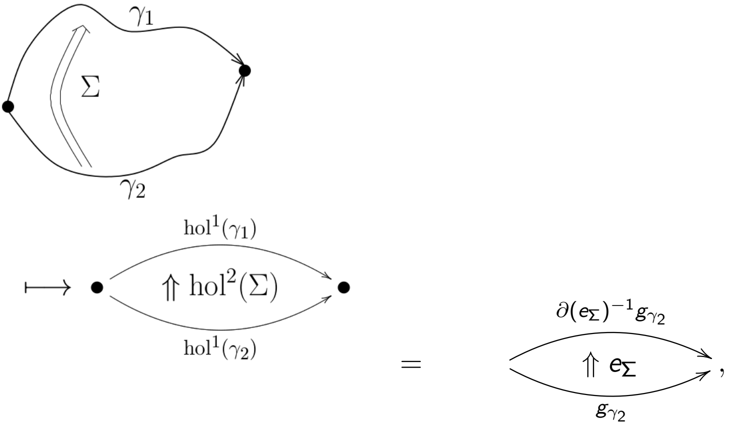

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Let $\mathcal{G} = (\partial\colon E \to G, \triangleright)$ be a crossed module.
Let M be a manifold. Let $L = (L^0, L^1, L^2, \dots)$ be a CW-decomposition of M (some minor "non-wildness" conditions).
A **discrete 2-connection** $\mathcal{F}$ is given by an assignment $\gamma \in L^1 \mapsto g_\gamma \in G$ and $P \in L^2 \mapsto e_P \in E$,
satisfying the **fake-flatness condition**:
If we have a configuration like:

Then: $\partial(e_P) = g_{\gamma_4}^{-1} g_{\gamma_3} g_{\gamma_2} g_{\gamma_1}$.

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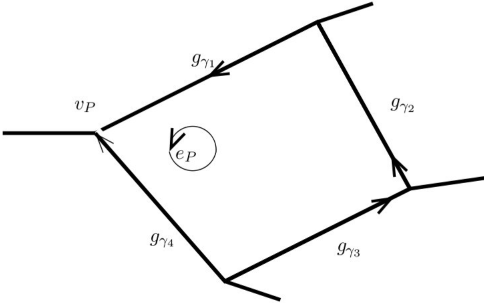

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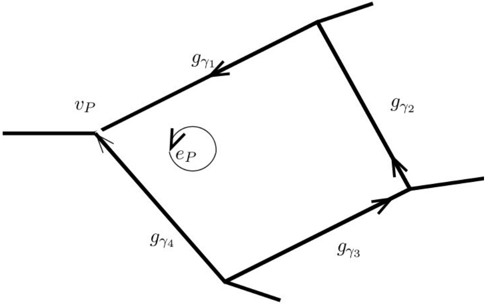

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The Hilbert space for the higher Kitaev model

- M a compact manifold of any dimension. Possibly with ∂.
- Then we put $\Phi(M, L) = \{Discrete\ 2 - connections\ \ \mathcal{F}\}$.
- And $V(M, L) = \mathbb{C}\Phi(M, L)$.
- The group of gauge operators puts together gauge transformations along vertices and along edges:

$$T(M, L) = \Big(\prod_{v \in L^0} G\Big) \ltimes \Big(\prod_{\sigma(t) \xrightarrow{t} \tau(t) \in L^1} E\Big)$$

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Discrete surface holonomy

Theorem: Let $\mathcal{F} \in \Phi(M, L)$ be a discrete 2-connection.

- Given a 2-sphere Σ cellularly embedded in M, and an initial point $v \in \Sigma$, we can define its 2-dimensional holonomy: $Hol_v^2(\mathcal{F}, \Sigma) \in \ker(\partial) \subset E$. arXiv:1702.00868

  This surface-holonomy depends only on the starting point $v \in \Sigma$, and not in the order whereby we combine 2-cells.

  If we change the base point then 2D holonomy changes by acting by a $g \in G$.

  For example, consider the discrete 2-connection on the tetrahedron Σ, below, based on the bottom left corner v_0.

Then $\mathrm{Hol}^2_{v_0}(\mathcal{F}, \Sigma) = e_1\ e_2^{-1}\ e_3^{-1}\ g_{12} \triangleright e_4$.

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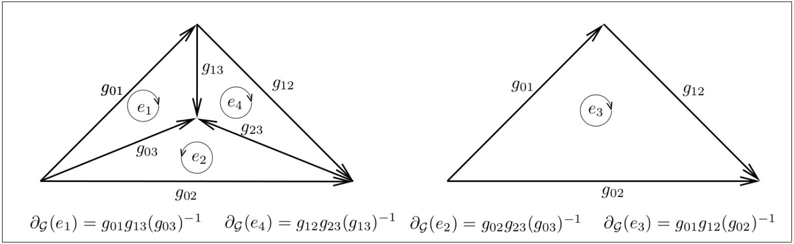

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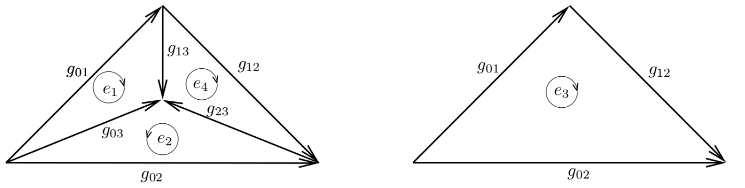

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Action of the group of gauge operators

- We have an action of the group of gauge operators $T(M, L)$ on $\Phi(M, L)$, preserving 2D holonomy, up to acting by G.

For edge operator, this action is defined from the 2D holonomy. Given $t \in L^1$, and $e \in E$, let U_t^e be the unique gauge operator supported in t with $U_t^e(t) = e$. (Called an edge gauge spike.) Given $v \in L^0$, and $g \in G$, let U_t^e be the unique gauge operator supported in v with $U_v^g(v) = g$. (Called a vertex gauge spike.)

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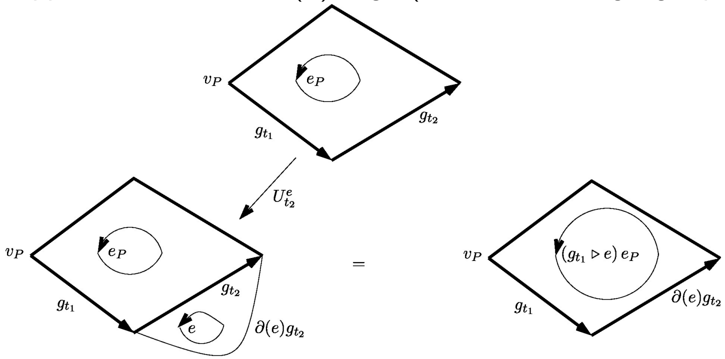

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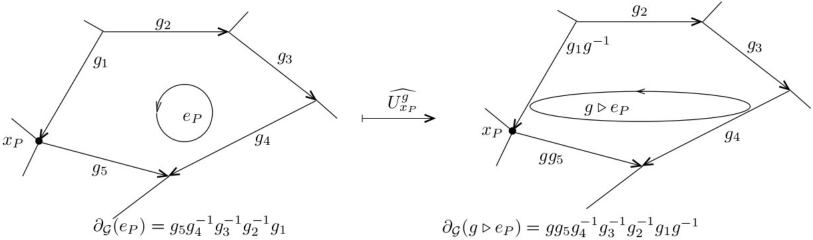

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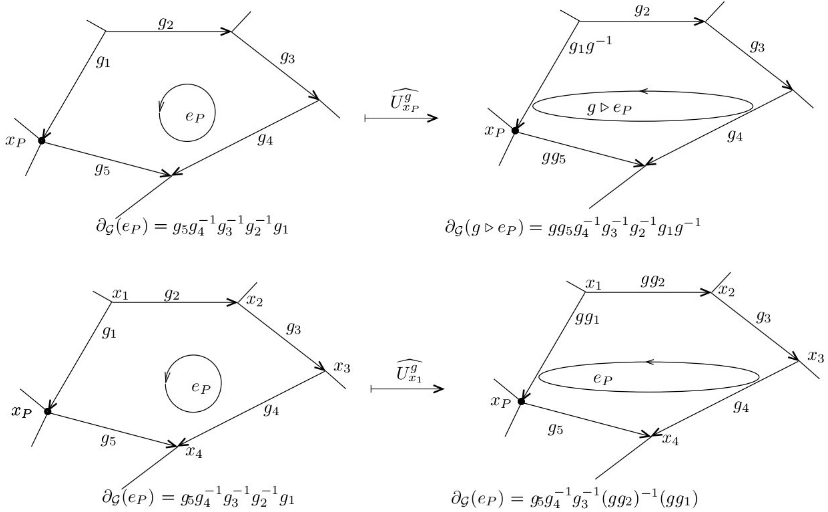

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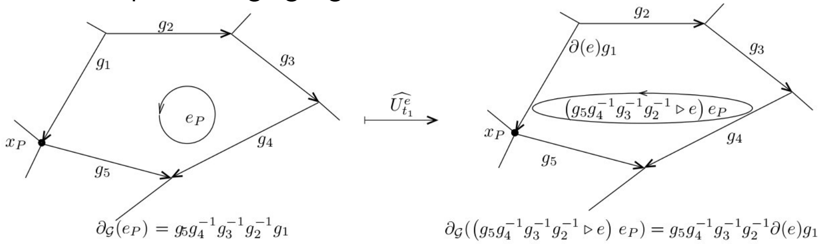

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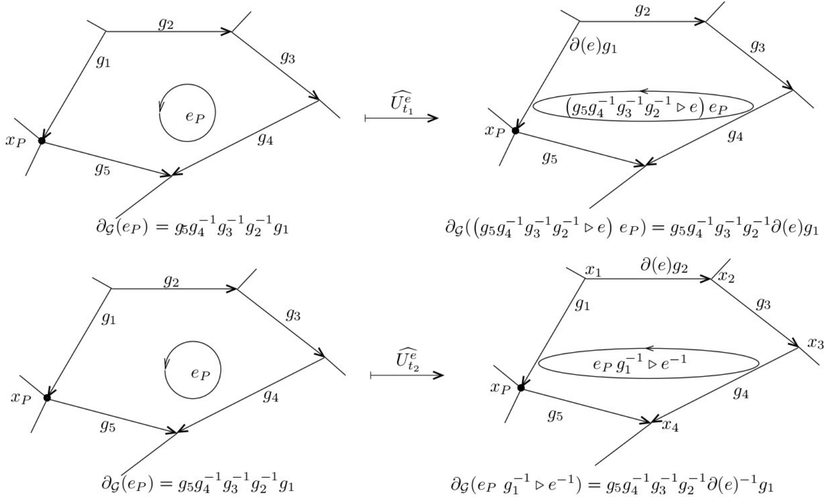

The higher Kitaev model

$V(M,L) = \mathbb{C}\{Discrete\ 2-connections\ \ \mathcal{F}\}$.
Hamiltonian $H\colon V(M,L) \to V(M,L)$.

$$H = -\frac{1}{|G|}\sum_{v\in L^0}\sum_{g\in G}\hat{U}_v^g - \frac{1}{|E|}\sum_{t\in L^1}\sum_{e\in E}\hat{U}_t^e - \sum_{b\in L^3}\mathcal{C}_b^{1_E}.$$

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Where $\mathcal{C}_b^k(\mathcal{F}) = \begin{cases}\mathcal{F}, \text{ if } 2hol(\mathcal{F},\partial b) = k\\ 0, \text{ otherwise}\end{cases}$, where $k \in \ker(\partial)$.
All operator in the last sum are commuting self-adjoint projectors.

$\mathcal{C}_b^{1_E}$ forces the surface-holonomy of a discrete 2-connection $\mathcal{F}$ to be trivial along the boundary of the 3-cell b.

Algebra generated by the U_t^g, U_t^e and C_b^k is our proposal for a local operator algebra. Relations are in arXiv:1702.00868.

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$$\Phi(M,L) = \{Discrete\ \ 2-connections\}.$$

Theorem

$$\Phi(M,L) \cong \hom(\Pi_2(M^2,M^1,M^0),\mathcal{G}), \quad \text{canonically}$$

Where $\Pi_2(M^2,M^1,M^0)$ is the fundamental crossed module of the filtered space (M^2,M^1,M^0), a crossed module of groupoids.

Theorem The ground state of $H\colon V(M,L) \to V(M,L)$ is
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As the geometric realisation of a simplicial set $B_{\mathcal{G}}$ has:

- one 0-simplex $\{*\}$
- One 1-simplex $* \xrightarrow{g} *$ for each $g \in G$.
- 2-simplices have the form (where $g, h \in G$ and $e \in E$):

- 3-simplices have the form
- n-simplices are analogously defined. Colourings of 1 and 2-cells of the n-simplex, fake-flat on trianges and flat on tetrahedra,

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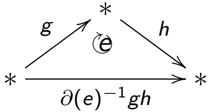

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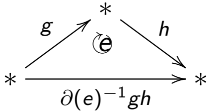

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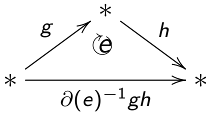

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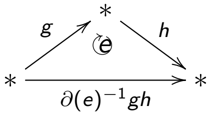

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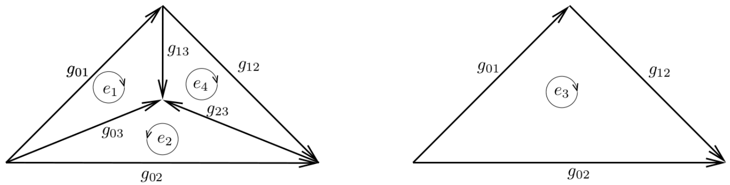

$$\partial_{\mathcal{G}}(e_1) = g_{01}g_{13}(g_{03})^{-1} \quad \partial_{\mathcal{G}}(e_4) = g_{12}g_{23}(g_{13})^{-1} \quad \partial_{\mathcal{G}}(e_2) = g_{02}g_{23}(g_{03})^{-1} \quad \partial_{\mathcal{G}}(e_3) = g_{01}g_{12}(g_{02})^{-1}$$

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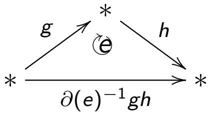

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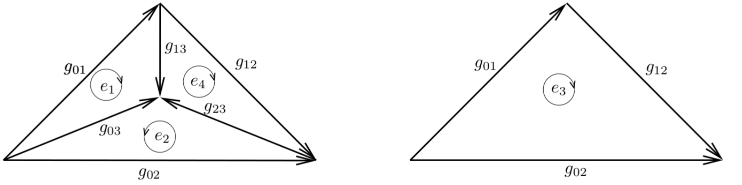

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Underpinning TQFT and invariants of loop braids

There is a (n+1)D TQFT whose state spaces $V(M)$, M an n-manifold, are the ground state of higher Kitaev over M: the Yetter homotopy 2-type TQFT. 1606.06639 + 1702.00868

Yetter TQFT computes homotopy cardinality of certain function spaces: math/0608484. Cf. Quinn total homotopy TQFT.

We stay in the homotopy language so:

$$V(M) = \mathbb{C}\{\textit{maps } f : M \to B_{\mathcal{G}}\}/\textit{homotopy}.$$

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Yetter TQFT yields invariants of 2-tangles $(C \xrightarrow{T} C') \subset D^4$.
C a link in $D^3 \times \{0\}$. C' a link in $D^3 \times \{1\}$.
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Diagrams as the one above generate the loop braid group LBG_2.
Given a 2-tangle $T : C \to C'$ we can consider its complement.
This yields a "pointed cobordism" of manifolds $W_T : M_C \to M_{C'}$.
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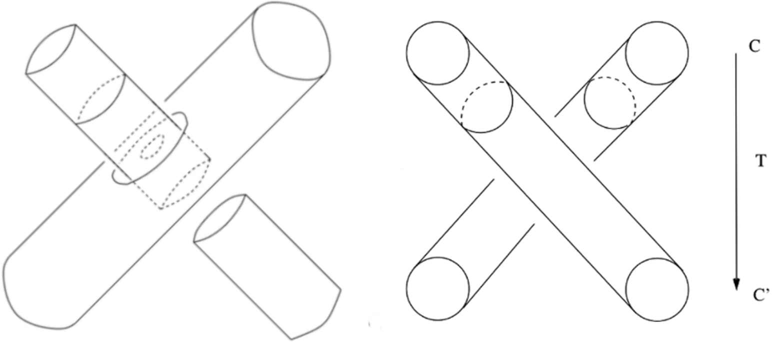

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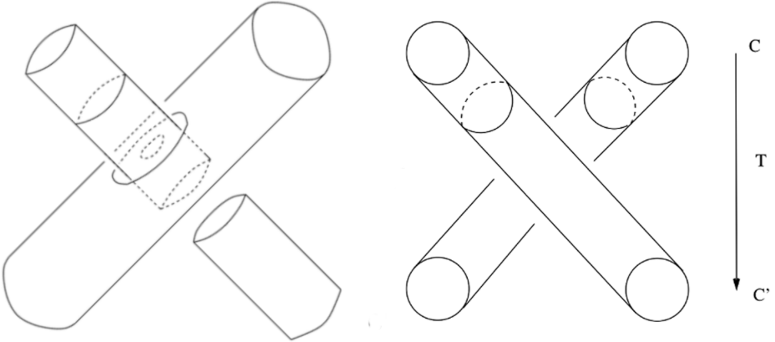

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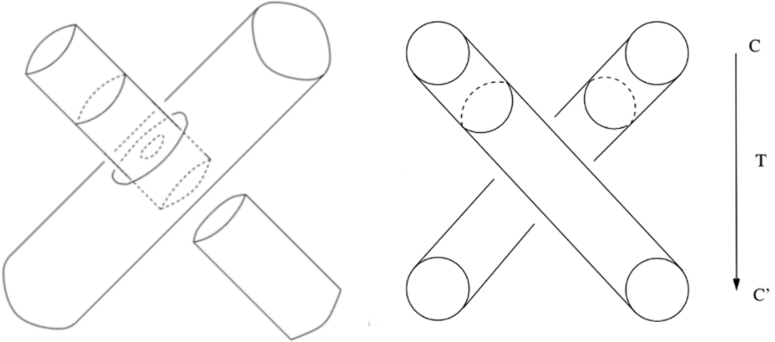

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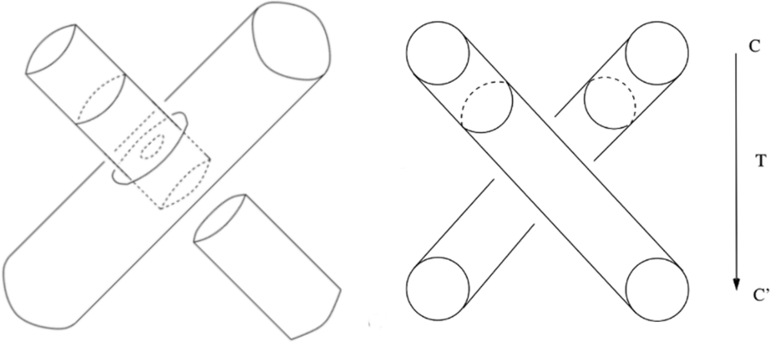

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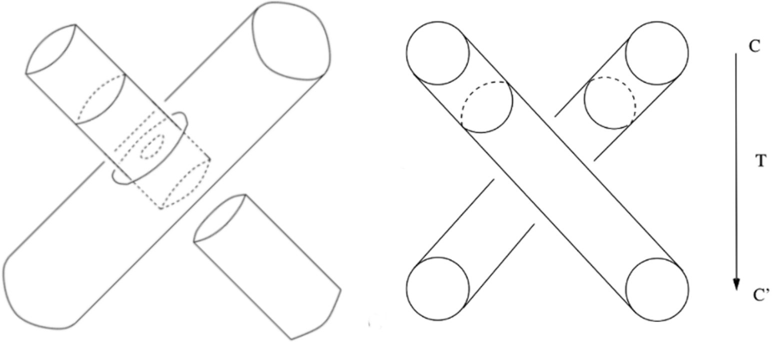

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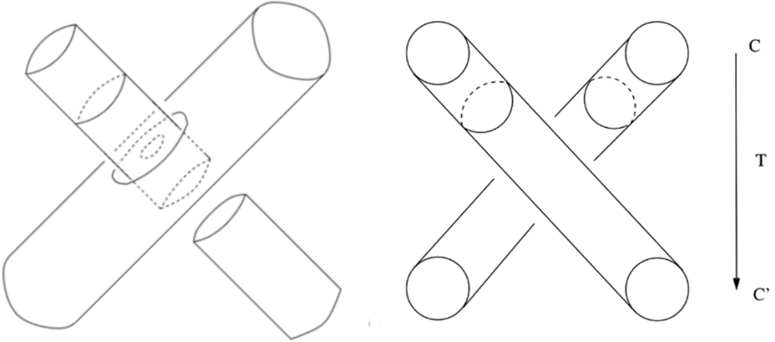

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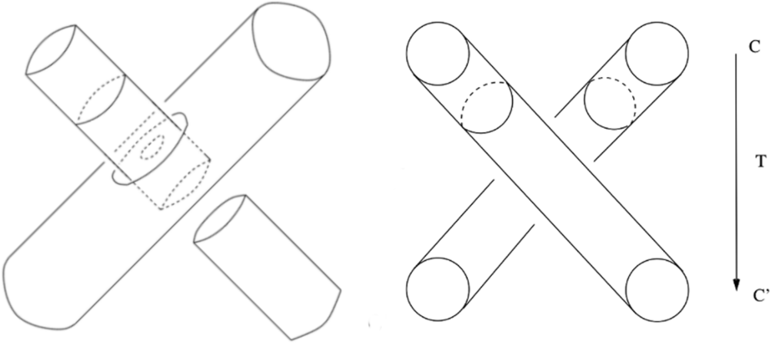
C a link in $D^3 \times \{0\}$. C' a link in $D^3 \times \{1\}$.
And $T \subset D^4 = [0,1]^4$ a surface connecting C and C'.



Diagrams as the one above generate the loop braid group LBG_2.
Given a 2-tangle $T\colon C \to C'$ we can consider its complement.
This yields a "pointed cobordism" of manifolds $W_T\colon M_C \to M_{C'}$.
Here $M_C = (D^3 \setminus C, *)$, $M_{C'} = (D^3 \setminus C, *)'$, $W_T = (D^4 \setminus T, * \times I)$.

The tube map: $\{Welded\ knots\} \to \{Loop\ braids\}$

Yetter / Quinn TQFT on complements loop braids (c.f. arXiv:0704.1246).

Via the tube map. $\{Welded\ \ braids\} \to \{Loop\ Braids\}$.

A welded braid.
(Diagrams are to be considered up to welded Reidemeister moves.)

The tube map in the vicinity of a classical and of a virtual crossing.

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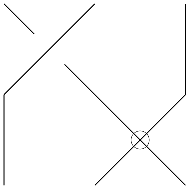

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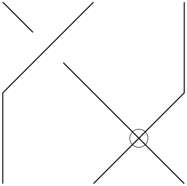

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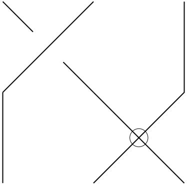

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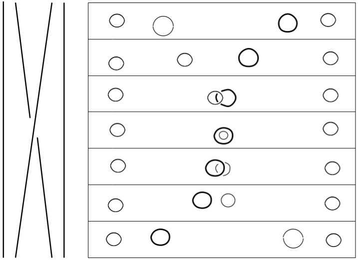

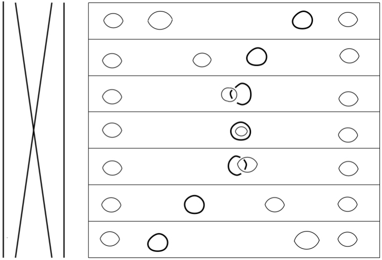

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Crossed module representations of loop braids

Let $\mathcal{G} = (\partial\colon E \to G, \triangleright)$ be a crossed module. We have a representation of the loop braid group WBG_n on $\mathbb{C}(G \times \ker \partial)^n$. It can be calculated by the following biquandle.

The extension to representations of the necklace braid group is quite do-able. (Though it has not been written down.)
And given the algebraic topological interpretation as maps to $B_{\mathcal{G}}$ one can also consider homology twistings via cocyles $\omega \in H^4(B_{\mathcal{G}}, U(1))$. Done for closed manifolds only. math/0608484

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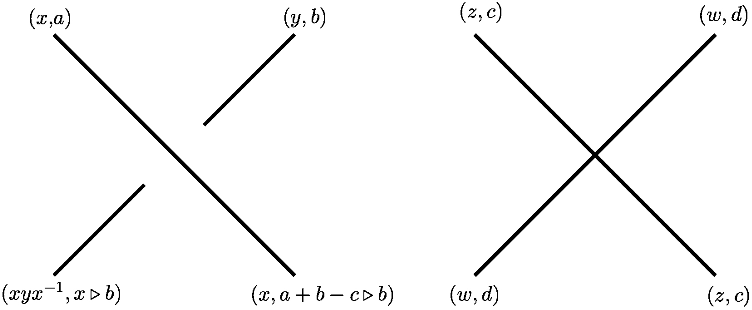

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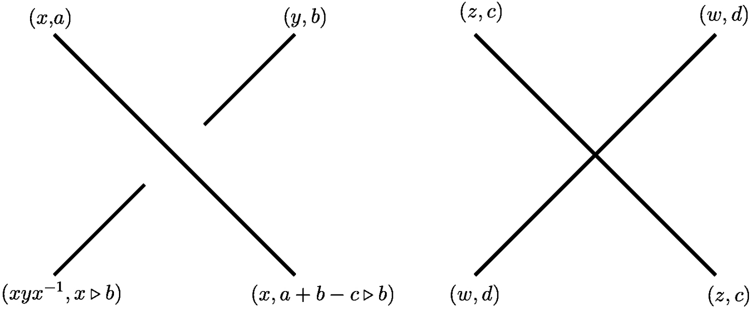

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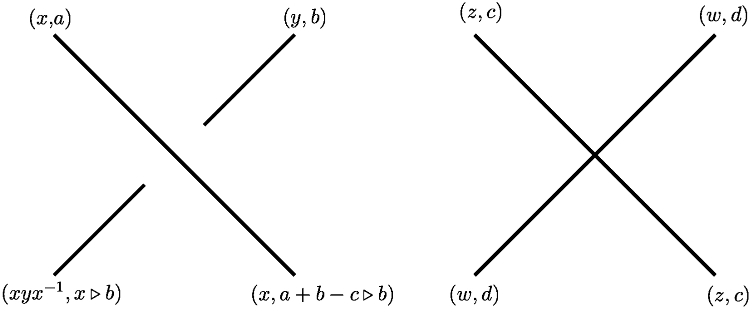

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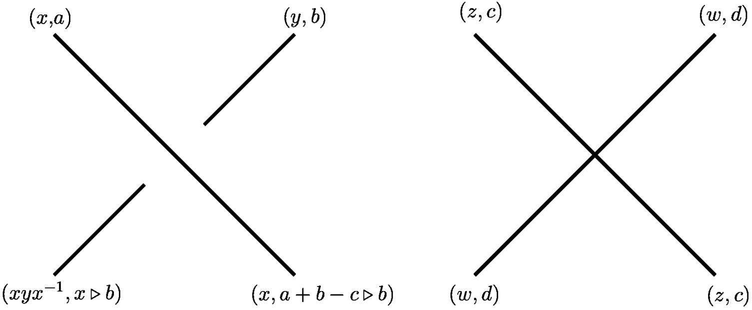

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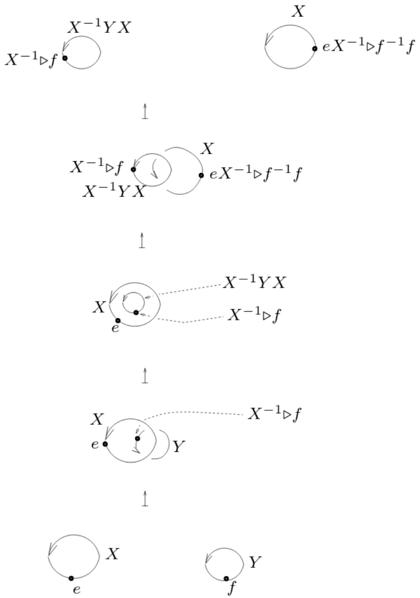

$X, Y \in G,\ e, f \in \ker(\partial)$.

Enrichments via operator algebras arXiv:1807.09551

Let $\mathcal{G} = (\partial\colon E \to G, \triangleright)$ be a crossed module.
Let Γ be the action groupoid of the conjugation action of $G \ltimes \ker(\partial)$ on itself.
Arrows of Γ have the form:

$$(x, a) \xrightarrow{(y,b)} (y, b) \triangleright (x, a) = \left(yxy^{-1}, b + y \triangleright a - (yxy^{-1}) \triangleright b\right).$$

Consider the groupoid algebra $\mathbb{C}(\Gamma)$ of Γ.

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$\overline{w}$ and $-\overline{w} \triangleright a$ are interpreted as the 2D holonomies of the tubes traced by each loop when they move. Aharonov-Bohm phases?

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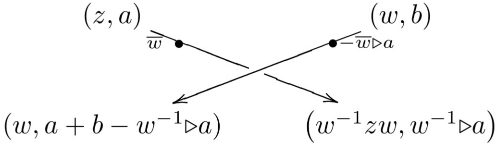

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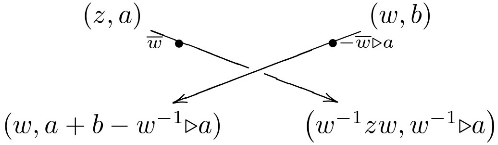

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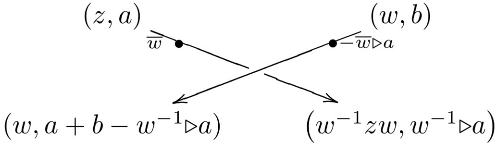

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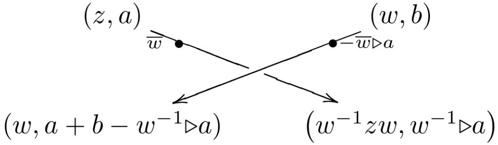

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Enrichments via operator algebras arXiv:1807.09551

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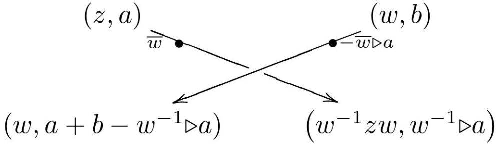

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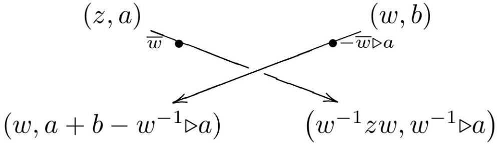

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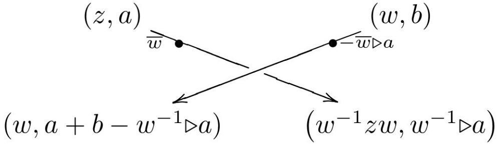

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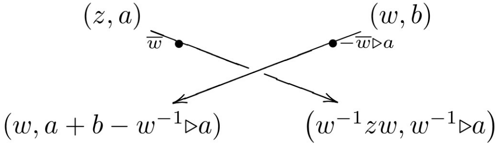

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