

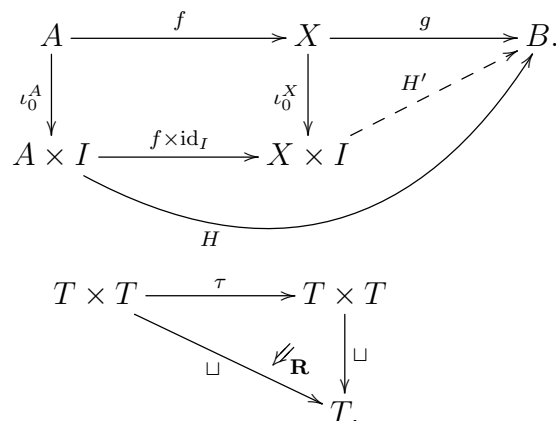
Sample HTML file: produced with lwrap, with mathematical formulae displayed with MathJax, and xymatrix commutative diagrams displayed with XyJax-v3

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The responsibility for this HTML file is João Faria Martins' only. Corrections, suggestions, etc. are very welcome, and can be sent to: j.fariamartins@leeds.ac.uk.

**This is strictly a non-specialist experimental file.**

See the links below for:

- “stand-alone” version of this webpage,
- “split” version of this webpage (with sans serif fonts),
- pdf version of this page,
- Latex source for the stand alone version of this page `Sample_Lwarp_Xy-jax.tex`,
- Latex source for the split version of this page, with sans serif fonts: `Sample_Lwarp_Xy-jax-split.tex`,
- CSS files for the stand-alone version of this page: `lwarp.css` (automatically provided by *lwarpmk*) and `my_file_stand-alone.css` (minor tweak to adjust margins),
- CSS files for the split version of this page: `lwarp_formal.css` (automatically provided by *lwarpmk*) and `my_file_split.css` (minor tweak enforcing sans serif fonts).
- Configuration file required for browsers to find Xy-Jaxv3, `lwarp-with-Xy-jax_v3.txt`. This script is essentially the original Lwarp MathJax emulation code, `lwarp_mathjax.txt` (automatically created by `lwarp v0.902`), with two minimal additions, as shown.

## 1 Context and instructions: using Lwarp to compile latex to HTML, so that xymatrix diagrams are displayed with Xy-Jax

This HTML file (that you are reading) was produced with the `lwarp.sty` package, available with **TeXLive**.<sup>1</sup> In particular the preamble contains:

```
\documentclass[a4paper,12pt]{article}
\usepackage[mathjax]{lwarp}
```

The version of the software was:

```
pdfTeX, Version 3.141592653-2.6-1.40.24 (TeX Live 2022) (preloaded format=pdflatex)
lwarpmk: lwarpmk: v0.910 Automated make for the LaTeX Lwarp package.
```

The source latex file can be found here: `Sample_Lwarp_Xy-jax.tex` or here `Sample_Lwarp_Xy-jax-split.tex`, depending whether you are seeing the “stand-alone” version of this webpage, or the “split” version of this webpage.

This HTML file also makes use of the following configuration script, so that **Xy-pic** diagrams, particularly *xymatrix* commutative diagrams, can be displayed by using **Xy-Jax-v3**. (Or at least a good sample of them: there seem to be some compatibility issues.)

`lwarp-with-Xy-jax_v3.txt`.

---

<sup>1</sup>See also <https://people.bath.ac.uk/feb/lwarp/lwarp-intro.html>, for a quick introduction on how to use Lwarp, and without which this explanation that you are reading would not have been written.

This script file has minimal additions (two in total) in the original Lwarp MathJax emulation code, that is automatically created by lwarp v0.902), namely:

lwarp\_mathjax.txt,

(Two lines were added to the script above, as shown, in order that browsers can find Xy-Jaxv3.) This modification was produced under items 5 and 6 of the licence <https://www.ctan.org/license/lpp11.3> (The LATEX Project Public License 1.3).

Credit is due to instructions in: <https://github.com/sonoisa/XyJax-v3> (the official Xy-Jaxv3 documentation) and in "40.11 lwarp\_mathjax.txt" of the official Lwarp documentation.

If using Xy-pic diagrams displayed with **XyJax-v3**, insert the following in the preamble of your latex file:

```
\usepackage[all]{xy}
\MathJaxFilename{lwarp-with-Xy-jax_v3.txt}
```

### Compile .tex to .html using:

1. *pdflatex Sample\_Lwarp\_Xy-jax.tex* (possibly twice) : creates Sample\_Lwarp\_Xy-jax.pdf (usual pdf file, that can also be shared) and Sample\_Lwarp\_Xy-jax\_html.tex. (And some other aux files.)
2. *lwarpmk html* : converts Sample\_Lwarp\_Xy-jax\_html.tex to Sample\_Lwarp\_Xy-jax\_html.html. And then creates Sample\_Lwarp\_Xy-jax.html (possibly split into different HTML files if such option is chosen).

**Note:** Use *lwarpmk html1* to force a recompile.

**Note:** In case of error, with *lwarpmk html*, use *lwarpmk pdftohtml* to create HTML file.

**Note:** When using *bibtex* files, then the following is required:

- *bibtex* Sample\_Lwarp\_Xy-jax\_html.aux
- *pdflatex* Sample\_Lwarp\_Xy-jax\_html.tex

The following steps are only needed once (any time tikz figures change). Only needed with tikz.

3. *lwarpmk limages* : If there are images done with tikz, tikz-cd, etc.
4. Repeat 2 (now using *lwarpmk html1*) and 3 (to finish creating the images).

**Note.** *It is advisable to use a recent version of **TeXLive**.*

### Warnings:

- The usage of **XyJax-v3** software with lwarp is experimental.
- Not all accessibility features of commutative diagrams displayed with XyJax-v3 work. Among the accessibility features that seem to work well are:
  1. View original latex source.
  2. Zoom maths expression.

## 2 Examples: xymatrix diagrams displayed with Xy-Jaxv3 using Lwarp

This is an example of a HTML file with:

- Mathematical formulae displayed with **MathJax**:

$$x^2 + y^2 = 1, \tag{1}$$

$$(x \xrightarrow{\gamma} y) \in \pi_1(X, X_0),$$

$$\cdots \rightarrow \pi_i(E_b, x) \xrightarrow{\iota} \pi_i(E, x) \xrightarrow{\partial} \pi_i(B, b) \xrightarrow{\delta} \pi_{i-1}(E_b, x) \rightarrow \cdots \xrightarrow{\iota} \pi_1(E, x) \xrightarrow{\partial} \pi_1(B, b) \xrightarrow{\delta_x} \pi_0(E_b) \xrightarrow{\iota} \pi_0(\dots)$$

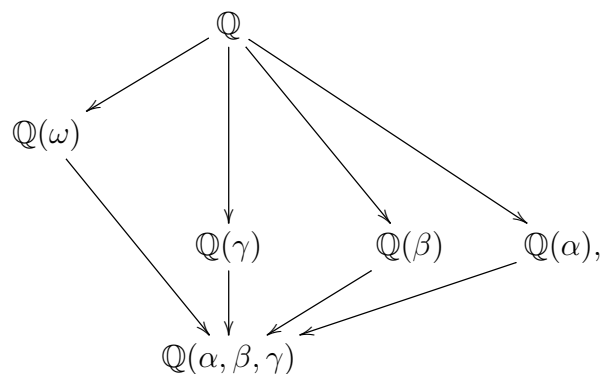
- To see how lecture notes look like with Lwarp, see subsections 6.1 and 6.2.
- This configuration is compatible with AMS-CD commutative diagrams, e.g. this diagram from the amscd package manual:

$$\begin{array}{ccc} A & \xrightarrow{a} & B \\ & \underset{b}{\downarrow} & \\ l \downarrow r & & l \uparrow r \\ C & \xleftarrow{a} & D \\ & \underset{b}{\downarrow} & \end{array}$$

- Commutative diagrams drawn with **Xy-pic** displayed by using **XyJax-v3**.  
For a good introduction to typesetting diagrams with Xy-pic see e.g.
  - [https://www.math.purdue.edu/~adebray/lecture\\_notes/using\\_xy.pdf](https://www.math.purdue.edu/~adebray/lecture_notes/using_xy.pdf) (Arun Debray Using Xy to typeset automata. July 5, 2016.)
  - <https://web.math.ku.dk/~holm/download/xyrefer.pdf> (Kristoffer H. Rose & Ross Moore: Xy-pic Reference Manual, Version 3.7 1999/02/16.)
  - See also the more recent user guide (Kristoffer H. Rose, Version 3.8.9, October 6, 2013) in the official page of xymatrix – Commutative diagrams using XY-pic

E.g.

$$\begin{array}{ccc} S_3 & \xrightarrow{f} & S_4 \\ & \searrow \cong & \downarrow \pi \\ & & S_4/V, \end{array}$$



(123).

$$\begin{array}{ccccc}
 A & \xrightarrow{f} & X & \xrightarrow{g} & B, \\
 \downarrow \iota_0^A & & \downarrow \iota_0^X & \nearrow H' & \\
 A \times I & \xrightarrow{f \times \text{id}_I} & X \times I & & \\
 & \searrow H & & & 
 \end{array}$$

$$\begin{array}{ccc}
 \bigoplus_{y \in H_0} \mathbf{H}_1(x, y) \otimes \mathbf{H}'_1(y, z) & \xrightarrow{\text{proj}} & \int^{y \in H_0} \mathbf{H}_1(x, y) \otimes \mathbf{H}'_1(y, z) \\
 \downarrow \bigoplus_{y \in H_0} \eta_{(x,y)} \otimes \eta'_{(y,z)} & & \downarrow \int^{y \in Y} \eta_{(x,y)} \otimes \eta'_{(y,z)} \\
 \bigoplus_{y \in H_0} \mathbf{H}_2(x, y) \otimes \mathbf{H}'_2(y, z) & \xrightarrow{\text{proj}} & \int^{y \in H_0} \mathbf{H}_2(x, y) \otimes \mathbf{H}'_2(y, z),
 \end{array}$$

$$\begin{array}{ccc}
 \int^{y \in Y} \overline{\mathbf{H}}_{(X,Y)}^{M_1}(-, y) \otimes \overline{\mathbf{H}}_{(Y,Z)}^{M_2}(y, -) & \xrightarrow{\eta_{(X,Y,Z)}^{M_1, M_2}} & \overline{\mathbf{H}}_{(X,Z)}^{M_1 \times_Y M_2} \\
 \parallel \mathbf{2H}_{(X,Y)}^{W_1} \bullet \mathbf{2H}_{(Y,Z)}^{W_2} & & \parallel \mathbf{2H}_{(X,Z)}^{W_1 \#_0 W_2} \\
 \int^{y \in Y} \overline{\mathbf{H}}_{(X,Y)}^{N_1}(-, y) \otimes \overline{\mathbf{H}}_{(Y,Z)}^{N_2}(y, -) & \xrightarrow{\eta_{(X,Y,Z)}^{N_1, N_2}} & \overline{\mathbf{H}}_{(X,Z)}^{N_1 \times_Y N_2}.
 \end{array}$$

$$\begin{array}{ccc}
 \mathcal{C} & \xrightarrow{F} & \mathcal{D} \\
 & \Downarrow \eta & \\
 \mathcal{C} & \xrightarrow{F'} & \mathcal{D}
 \end{array}$$

$$\begin{array}{ccc}
 T \times T & \xrightarrow{\tau} & T \times T \\
 & \searrow \sqcup & \downarrow \sqcup \\
 & & T.
 \end{array}$$

$$\begin{array}{ccc}
 \mathcal{C} \sqcup \mathcal{C}' & \xrightarrow{T_b \circ \langle f, f' \rangle} & \text{Grp.} \\
 & \Downarrow \langle \eta, \eta' \rangle & \\
 \mathcal{C} \sqcup \mathcal{C}' & \xrightarrow{\langle T, T' \rangle} & \text{Grp.}
 \end{array}$$

$$\begin{array}{ccc}
 \begin{array}{ccc}
 \mathcal{C} & \xrightarrow{F} & \mathcal{D} \\
 & \Downarrow \eta & \\
 \mathcal{C} & \xrightarrow{F''} & \mathcal{D} \\
 & \Downarrow \eta' & \\
 \mathcal{C} & \xrightarrow{F''} & \mathcal{D}
 \end{array} & = & \begin{array}{ccc}
 \mathcal{C} & \xrightarrow{F} & \mathcal{D} \\
 & \Downarrow \eta \#_1 \eta' & \\
 \mathcal{C} & \xrightarrow{F''} & \mathcal{D}
 \end{array}
 \end{array}$$

$$\begin{array}{ccccc}
(\mathbf{Cob}^{(n,n+1,n+2)})^3 & \xrightarrow{\mathcal{F}_B \times \mathcal{F}_B \times \mathcal{F}_B} & & & (\mathbf{Prof})^3 \\
\downarrow \text{id} \times \sqcup & \searrow \sqcup \times \text{id} & \Downarrow \chi' \times \mathcal{F}_B & & \downarrow \otimes \times \text{id} \\
(\mathbf{Cob}^{(n,n+1,n+2)})^2 & \xleftarrow{\alpha} & (\mathbf{Cob}^{(n,n+1,n+2)})^2 & \xrightarrow{\mathcal{F}_B \times \mathcal{F}_B} & (\mathbf{Prof})^2 \\
\downarrow \sqcup & \swarrow \sqcup & \swarrow \chi' & \searrow \otimes & \downarrow \otimes \times \text{id} \\
(\mathbf{Cob}^{(n,n+1,n+2)}) & \xrightarrow{\mathcal{F}_B} & \mathbf{Prof} & & (\mathbf{Prof})^2 \\
& & & & \swarrow \otimes \\
& & & & \mathbf{Prof}
\end{array}$$

$\xRightarrow{\omega'}$

$$\begin{array}{ccccc}
(\mathbf{Cob}^{(n,n+1,n+2)})^3 & \xrightarrow{\mathcal{F}_B \times \mathcal{F}_B \times \mathcal{F}_B} & & & (\mathbf{Prof})^3 \\
\downarrow \text{id} \times \sqcup & & \Downarrow \mathcal{F}_B \times \chi' & \swarrow \text{id} \times \otimes & \downarrow \otimes \times \text{id} \\
(\mathbf{Cob}^{(n,n+1,n+2)})^2 & \xrightarrow{\mathcal{F}_B \times \mathcal{F}_B} & (\mathbf{Prof})^2 & \swarrow \alpha & (\mathbf{Prof})^2 \\
\downarrow \sqcup & \Downarrow \chi' & \downarrow \otimes & \swarrow \otimes & \downarrow \otimes \times \text{id} \\
(\mathbf{Cob}^{(n,n+1,n+2)}) & \xrightarrow{\mathcal{F}_B} & \mathbf{Prof} & & (\mathbf{Prof})^2 \\
& & & & \swarrow \otimes \\
& & & & \mathbf{Prof}
\end{array}$$

### 3 Issues when using XyJax-v3 and Lwarp for typesetting xymatrix diagrams

Please do contact me if you know the solution to any of these issues. Remember: this is strictly a non-specialist file!

- The commands of the form:

`\stackrel{(*)}{\implies}`

or

`\left ( x \xrightarrow{f} y \right)`

do not seem to work well inside

`\begin{equation} \end{equation}.`

So use instead, for example,

`$$ \stackrel{(*)}{\implies} $$`

`$$ \left ( x \xrightarrow{f} y \right) $$`

. Here is how it looks:

$$\begin{array}{c}
(*) \\
\implies \\
\left( x \xrightarrow{f} y \right)
\end{array}$$

- Sometimes there are errors if there are mathematical commands in the title of sections, subsections, etc.
- Lwarpmk gives error messages if it finds *xymatrix* inside:

```
\begin{equation} \end{equation}.
```

(However a html file can still be produced with *lwarpmk pdftohtml*.)

So avoid:

```
\begin{equation} \xymatrix{ } \end{equation},
```

and use instead:

```
\[ \xymatrix{ } \],
```

or

```
$$ \xymatrix{ } $$.
```

- **Not all accessibility functions of MathJax work with Xy-Jaxv3, only some can be used, e.g. zoom and source latex code.**  
Accessibility features of Xy-jax are not officially supported.
- References to section, chapters etc, have glitches **if inside mathematics environments**.  
For instance, the links provided do not appear to work well. E.g.

$$\exp(x + yi) = e^x (\cos(y) + i \sin(y)) \quad (2)$$

$e^{i\pi} + 1 = (\cos(\pi) + i \sin(\pi)) + 1$ , using Equation (2). (Just for testing) equations (1), (3)  
= 0. A reference to second section 2

Outside maths environments all seems to work fine:

Using Equation (2). (Just for testing) equations (1), (3)  
A reference to second section 2

- When compiling xymatrix, or tikz, diagrams as sgv figures, with Alt-Text (which sometimes is necessary) it sometimes happens that the figure is not cropped correctly. This seems to depend strongly on the operating system being used.

## 4 More general notes

- **pdf** and **HTML** automatically coexist: here is the pdf version of this HTML file `Sample_Lwarp_Xy-jax.pdf`
- It is possible to split an HTML page into sub-pages: credit <https://people.bath.ac.uk/feb/lwarp/lwarp-intro.html>. Instructions can be seen in the source latex file of this HTML file `Sample_Lwarp_Xy-jax.tex`.

- Additions are required in the .tex file in order that **MathJax** displays commands, macros and definitions correctly. E.g. write:

```
\DeclareMathOperator{\Sym}{Sym}
\def \Mon {{\mathbf{Mon}}}
\newcommand{\Q}{\mathbb{Q}}
```

and then:

```
\begin{warpHTML}
  \CustomizeMathJax{\DeclareMathOperator{\Sym}{Sym}}
  \CustomizeMathJax{\def \Mon {{\mathbf{Mon}}}}
  \CustomizeMathJax{\newcommand{\Q}{\mathbb{Q}}}
\end{warpHTML}
```

This means that you can use slightly different versions of commands for pdf and for html.

- If using Xy-pic diagrams (if displayed as Xy-jax) put in the preamble of the tex file:

```
\MathJaxFilename{lwrap-with-Xy-jax_v3.txt}
```

You will also need this configuration file: lwrap-with-Xy-jax\_v3.txt. (Explanation is in Section 1.)

- Sample use of Xy-pic compatible with lwrap (so that the resulting xymatrix code is readable by Xy-Jax): E.g.:

```
\[
\begin{matrix}
& \mathbb{Q} & & \mathbb{Q}(\omega) & & \\
& \mathbb{Q}(\gamma) & \mathbb{Q}(\beta) & \mathbb{Q}(\alpha) & & \\
& \mathbb{Q}(\alpha, \beta, \gamma) & & & & 
\end{matrix}
\]
```

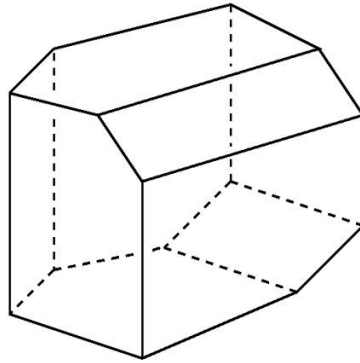
or

```
\displaymathnormal{
\begin{matrix}
& S_3 & & S_4 & & \\
& \mathbb{Q}(\gamma) & \mathbb{Q}(\beta) & \mathbb{Q}(\alpha) & & \\
& \mathbb{Q}(\alpha, \beta, \gamma) & & & & 
\end{matrix}
}
```

- Figures, with alternative text, can be included like this:

```
\includegraphics[width = 0.4\textwidth,
alt={Type your alternative text here}]{Stasheff.png}
```





- Figures, including **tikz** and **tikz-cd** figures, can be compiled as figures with **Alt Text**. (The same option is also available for xymatrix diagrams, and may be preferable to Xy-Jaxv3, in the cases when it is sensible to provide a comprehensive alternative text to the figure/diagram, instead of relying mainly on the availability of the latex code, for accessibility.)

**Warning:** In some operating systems, issues seem to exist with the conversion of tikz figures to svg: e.g. figures may be incorrectly cropped for the web-page. Size of xymatrix figures and tikz-cd figures seemingly then must be adjusted manually.

- When using **tikz** pictures use (note the Alt Text option):

```
\begin{figure}\ThisAltText{Alt text to your diagram}
\begin{tikzpicture}

\end{tikzpicture}
\end{figure}
```

or

```
\begin{center}\ThisAltText{Alt text to your diagram}
\begin{tikzpicture}

\end{tikzpicture}
\end{center}
```

Examples can be found towards the end of this file.

- When using **tikz-cd** use:

```
{\displaymathother
\ThisAltText{ Alt text to your commutative diagram }
\[
\begin{tikzcd} F(A) \arrow{r}{F(f)} & F(B) \\
G(A) \arrow{r}[swap]{G(f)} & \arrow{u}{\eta_A} & G(B) \\
& \arrow{u}[right]{\eta_B} & \\
\end{tikzcd}
\]
```

- It is possible to compile an **Xy-pic** diagram as a figure with alt text. Use:

```
{
\displaymathother
\ThisAltText{Write some alt text here.}
$$ \xymatrix{ } $$
}
```

Examples can be found towards the end of this file.

- **To select your own .CSS files use:**

```
\CSSFilename{your_file.css}
```

- Avoid good old Tex commands like

```
{\bf }, {\it }.
```

It seems that lwrap does not deal with them properly. Instead use:

```
\textbf{ }, \textit{ }.
```

(In fact better to avoid *italics* altogether for accessibility reasons: use **bold**.)

- Lwrap gives errors messages when compiling Xy-pic diagrams inside

```
\begin{equation} \end{equation}
```

Use instead:

```
\[\xymatrix{ } \]
or
\[\begin{xy} \xymatrix{ } \end{xy}\]
```

Or use *lwrapmk pdftohtml* if there are compilation errors.

- Figures created with tikz and xymatrix frequently have issues: e.g. they may be too small (so size needs to be adjusted), or incorrectly cropped. Seems to depend on operating system.

## 5 Some examples of tikz and xymatrix diagrams compiled as svg images, with alternative text

As written in Section 2, it is possible to display xymatrix diagrams by using XyJax-v3. In some cases, it may however be preferable to compile xymatrix diagrams as figures with **Alt Text**. (E.g. it may be sensible to instead provide a comprehensive alternative text to the figure/diagram, instead of relying mainly on the availability of the latex code, for accessibility.)

This option (svg image with alternative text) is also available for tikz and tikz-cd figures. Examples are below.

### 5.1 An xymatrix diagram compiled as an image with alt text

An *xymatrix* diagram compiled into a picture with alt text. Can utilize equation numbers, with no errors. The size of the svg image must be adjusted. Sometimes the image is not correctly cropped: depends on operating system.

$$\begin{array}{ccc} T \times T & \xrightarrow{\tau} & T \times T \\ & \searrow & \downarrow \sqcup \\ & & T \end{array} \quad (3)$$

### 5.2 A tikz image compiled as an image with alt text:

Size can be adjusted. Sometimes (depending on operating system, it seems) the image may be incorrectly cropped. See figure 1.

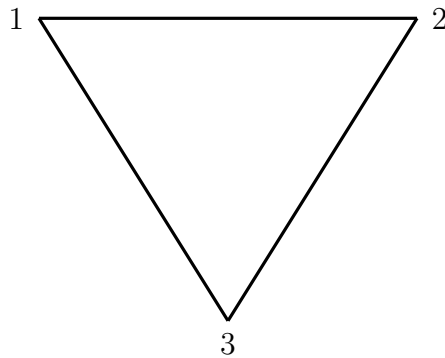
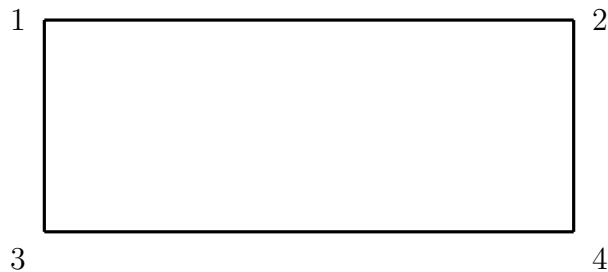


Figure 1: An equilateral triangle with its vertices labeled 1, 2 and 3

### 5.3 A tikz image compiled to an image with alt text, outside a float environment



## 5.4 A tikz image compiled to an image with alt text, outside a float environment, and with a caption

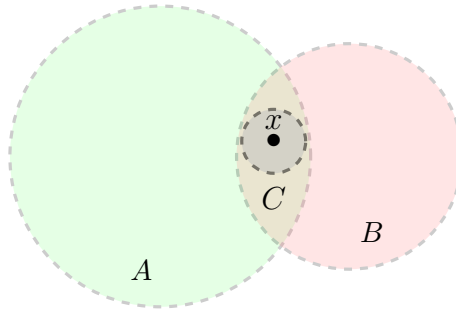


Figure 2: Given any  $A, B \in \mathcal{F}$ , if  $x \in A \cap B$  there exists  $C \in \mathcal{F}$  such that  $x \in C \subseteq A \cap B$ .

## 5.5 A tikz-cd diagram compiled into a figure with alt text:

Issues with size of diagrams created with tikz-cd. Sometimes (depending on operating system) the image may be incorrectly cropped.

The following diagram commutes.

$$\begin{array}{ccc}
 F(A) & \xrightarrow{F(f)} & F(B) \\
 \eta_A \uparrow & & \uparrow \eta_B \\
 G(A) & \xrightarrow{G(f)} & G(B)
 \end{array}$$

## 6 Sample mathematics

### 6.1 Adjoint functors and coadjoint functors via universal arrows

Let  $\mathcal{C}$  and  $\mathcal{D}$  be categories. Let  $G: \mathcal{D} \rightarrow \mathcal{C}$  be a functor.

**Definition 1.** Let  $A$  be an object of  $\mathcal{C}$ . A **universal arrow** from  $A$  to  $G: \mathcal{D} \rightarrow \mathcal{C}$  is a pair:

$$\left( F_A, A \xrightarrow{\eta_A} G(F_A) \right),$$

where  $F_A$  is an object in  $\mathcal{D}$  and  $\eta_A: A \rightarrow G(F_A)$  is an arrow in  $\mathcal{C}$ , such that the following universal property is satisfied:

For any object of  $B$  of  $\mathcal{D}$  and any arrow  $f: A \rightarrow G(B)$ , in  $\mathcal{C}$ , there exists a **unique** arrow  $\hat{f}: F_A \rightarrow B$ , in  $\mathcal{D}$ , that makes the following diagram, in  $\mathcal{C}$ , commute:

$$\begin{array}{ccc}
 A & \xrightarrow{\eta_A} & G(F_A) \\
 & \searrow f & \downarrow G(\hat{f}) \\
 & & G(B)
 \end{array}$$

**Exercise 2.** In the conditions of the previous definition, prove that if  $(F_A, A \xrightarrow{\eta_A} G(F_A))$  is a universal arrow from  $A$  to  $G$ , then we have a bijection:

$$\phi_{A,B}: \text{hom}_{\mathcal{C}}(A, G(B)) \longrightarrow \text{hom}_{\mathcal{D}}(F_A, B),$$

such that

$$(f: A \rightarrow G(B)) \xrightarrow{\phi_{A,B}} (\hat{f}: F_A \rightarrow B).$$

Moreover, prove that the bijection  $\phi_{A,B}$  is natural in  $B$ . This means that given any arrow  $g: B \rightarrow C$ , in  $\mathcal{D}$ , the following diagram (in the category of sets) commutes:

$$\begin{array}{ccc} \text{hom}_{\mathcal{C}}(A, G(B)) & \xrightarrow{\phi_{A,B}} & \text{hom}_{\mathcal{D}}(F_A, B) \\ m \mapsto G(g) \circ m \downarrow & & \downarrow n \mapsto g \circ n \\ \text{hom}_{\mathcal{C}}(A, G(C)) & \xrightarrow{\phi_{A,C}} & \text{hom}_{\mathcal{D}}(F_A, C) \end{array}$$

## 6.2 The Galois correspondence for $f(t) = t^3 - 2$ , over the rational field

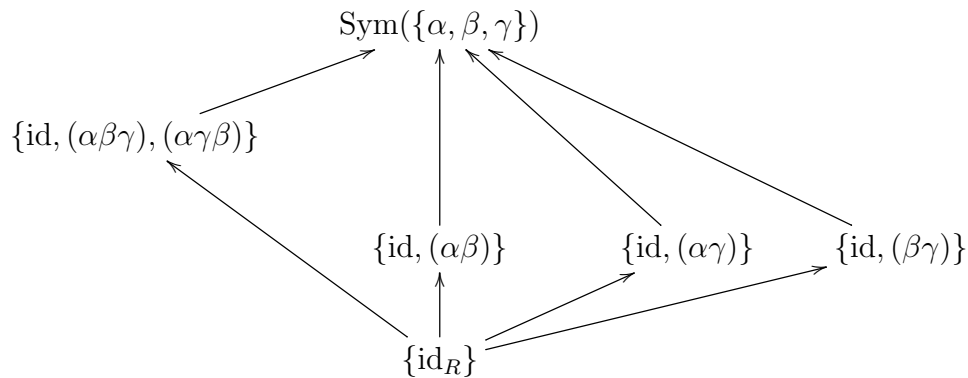
Let  $f(t) = t^3 - 2 \in \mathbb{Q}[t]$ . Let  $\omega = e^{\frac{2\pi i}{3}}$ . Let  $\alpha = \sqrt[3]{2}$ ,  $\beta = \alpha\omega$  and  $\gamma = \alpha\omega^2$ . Hence the set of roots of  $f$  is  $\{\alpha, \beta, \gamma\}$ . The splitting field of  $f$ , over  $\mathbb{Q}$  is:

$$\mathbb{Q}(\alpha, \beta, \gamma) = \mathbb{Q}(\alpha, \omega).$$

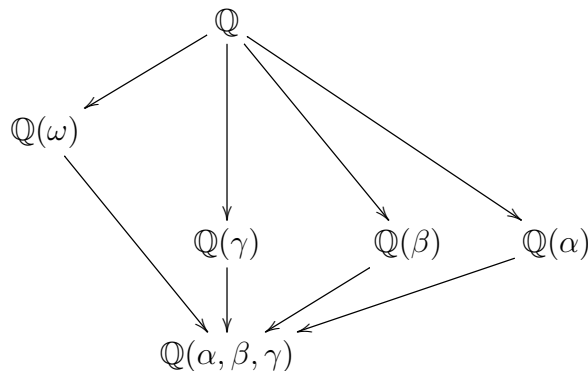
The monomorphism of groups,

$$\begin{aligned} \theta: \Gamma(f, \mathbb{Q}) &\longrightarrow \text{Sym}(\{\alpha, \beta, \gamma\}) \\ \tau &\longmapsto \left( \begin{array}{l} \{\alpha, \beta, \gamma\} \longrightarrow \{\alpha, \beta, \gamma\} \\ a \longmapsto \tau(a) \end{array} \right), \end{aligned}$$

is, in this case, an isomorphism. The diagram of subgroups of  $\Gamma(f, \mathbb{Q}) \cong \text{Sym}(\{\alpha, \beta, \gamma\})$  is below (note that inclusions go in the direction of arrows):



The corresponding diagram of intermediate fields  $\mathbb{Q} \subseteq L \subseteq \mathbb{Q}(\alpha, \beta, \gamma)$  is:



Also note that  $\{\text{id}_R\}$ ,  $\text{Sym}(R)$  and  $\{\text{id}, (\alpha\beta\gamma), (\alpha\gamma\beta)\} \cong A_3$  are the only normal subgroups of  $\text{Sym}(R)$ . So, respectively,

$$\mathbb{Q}(\alpha, \beta, \gamma) : \mathbb{Q}, \quad \mathbb{Q} : \mathbb{Q} \quad \text{and} \quad \mathbb{Q}(\omega) : \mathbb{Q} \quad (4)$$

are the only normal extensions of  $\mathbb{Q}$  contained in  $\mathbb{Q}(\alpha, \beta, \gamma)$ . Note that the fact that the extensions in (4) are normal also follows from the fact that they are the splitting fields of  $p(t) = t^3 - 2$ ,  $q(t) = t$  and  $r(t) = t^2 + t + 1$  (which has  $\omega$  and  $\omega^2$  as roots), over  $\mathbb{Q}$ .

**Remark 3.** Also note that we have a series of subfields of  $\mathbb{C}$  (we use  $\leq$  to denote subfield):

$$\mathbb{Q} \leq \mathbb{Q}(\omega) \leq \mathbb{Q}(\alpha, \beta, \gamma) = \mathbb{Q}(\alpha, \omega).$$

Note that:

- $\mathbb{Q}(\omega) : \mathbb{Q}$  is a normal extension (since it is the splitting field of  $t^2 + t + 1$ , over  $\mathbb{Q}$ ).
- $\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)$  is also a normal extension. This is because  $\mathbb{Q}(\omega, \alpha)$  is the splitting field of  $t^3 - 2$  over  $\mathbb{Q}(\omega)$ .

And then it follows that:

- $\Gamma(\mathbb{Q}(\omega, \alpha), \mathbb{Q}(\omega))$  is normal in  $\Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q})$ ,
- we have a series of subgroups of  $\Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q})$ :

$$\{e\} = \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega, \alpha)) \trianglelefteq \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)) \trianglelefteq \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}),$$

- The quotient groups can be explicitly determined:

$$- \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}) / \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)) \cong \Gamma(\mathbb{Q}(\omega) : \mathbb{Q}) \cong \mathbb{Z}_2.$$

Where the last equation follows since  $\mathbb{Q}(\omega) : \mathbb{Q}$  is a normal extension of degree 2, since it is the splitting field of the irreducible polynomial  $t^2 + t + 1$ , over  $\mathbb{Q}$ . This  $\mathbb{Z}_2$  is an abelian group.

$$- \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)) / \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega, \alpha)) \cong \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)) \cong \mathbb{Z}_3.$$

Where the last equation follows since  $\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)$  is a normal extension of degree 3. This is because it is the splitting field of the polynomial  $t^3 - 2$ , which is irreducible over  $\mathbb{Q}(\omega)$ . This is an abelian group.

What was just shown is a general pattern that exists any time we compute the splitting field of a polynomial that is soluble by radicals.