TQFTS and models for topological phases derived from categorified gauge theory (higher gauge theory)

I ENCONTRO BRASILEIRO EM TEORIA DAS CATEGORIAS

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The category of manifold and cobordisms (sketch)

Consider the (symmmetric monoidal) category $(\mathbf{n}, \mathbf{n+1})\text{-}\mathrm{Cob}$.

- Objects: $(n-1)$-compact manifolds $A, B, \ldots$
- Morphisms $[M]\colon A \to B$ are equivalence classes of diagrams:

$$A \xrightarrow{\;i\;} M \xleftarrow{\;j\;} B\,.$$

Where M is a smooth $(n-1)$-manifold, and
i and j induce a diffeomorphism $\langle i, j \rangle \colon A \sqcup B \to \partial(M)$.

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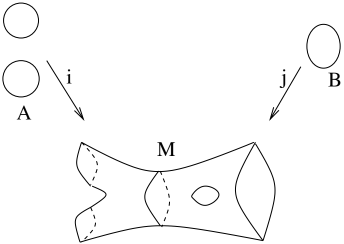

Visualization.

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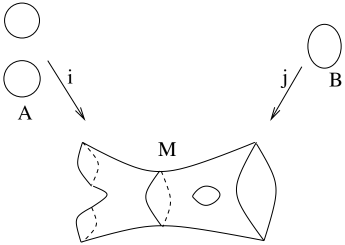

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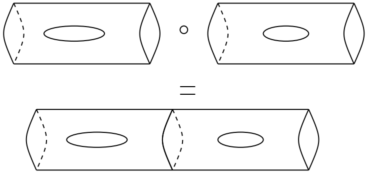

Composition of morphisms
Issues with smooth structure.

Topological quantum field theories

Definition

Given a non-negative integer n,
a Topological Quantum Field Theory (TQFT)
is a symmetric monoidal functor:

$$\mathcal{F}\colon (\mathbf{n}, \mathbf{n+1})\text{-Cob} \to \text{Vect}$$

In this talk I will:

- Recall Quinn's total homotopy TQFT
  $\mathcal{F}_{\mathbb{B}}^{(s)}\colon (\mathbf{n}, \mathbf{n+1})\text{-Cob} \to \text{Vect}$
  (Here $\mathbb{B}$ a homotopically finite space: a parameter of theory)
- Explain combinatorial calculation of $\mathcal{F}_{\mathbb{B}}^{(s)}$
  if $\mathbb{B}$ is the classifying space of a homotopy finite ω-groupoid.
- Relate to higher gauge theory.
- In passing mention higher Kitaev models; cf. Teotónio's talk.

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Homotopy finite spaces (or simplicial sets)

Definition

A space X is homotopy finite (HF) if:

- X has only a finite number of path components.
- If $K \in \pi_0(X)$ – set of path components of X – then $\pi_i(K)$ is:
  - trivial if $i > n$, for some n.
  - finite for all i.

If X is HF, the homotopy content of X is:

$$\chi^\pi = \sum_{K \in \pi_0} \frac{|\pi_2(K)|\ |\pi_4(K)|\ |\pi_6(K)| \dots}{|\pi_1(K)|\ |\pi_3(K)|\ |\pi_5(K)| \dots} \in \mathbb{Q}$$

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Notes about homotopically finite spaces

- If X and Y are HF then so are $X \times Y$ and $X \sqcup Y$, and:

$$\chi^\pi(X \times Y) = \chi^\pi(X) \times \chi^\pi(Y)$$

$$\chi^\pi(X \sqcup Y) = \chi^\pi(X) + \chi^\pi(Y)$$

- If $p \colon E \to B$ is a (Hurewicz) fibration of HF spaces
  B path-connected, $b \in B$, $F_b = p^{-1}(b)$:

$$\chi^\pi(E) = \chi^\pi(B) \times \chi^\pi(F_b)$$

- If M is a compact CW-complex, $\mathbb{B}$ is HF space.
  Then the function space below is HF

$$\mathrm{TOP}(M, \mathbb{B}) = \{f : M \to \mathbb{B} \mid f \text{ is continuous}\}$$

  In particular if M is a compact smooth manifold.

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Let $\mathbb{B}$ be a HF-space. Let $s \in \mathbb{C}$.

We define a functor: $\mathcal{F}_{\mathbb{B}}^{(s)} : (\mathbf{n}, \mathbf{n+1})\text{-Cob} \to \mathrm{Vect}$

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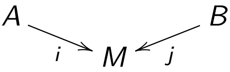

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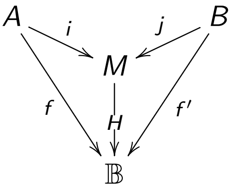

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Quinn TQFT $\mathcal{F}_{\mathbb{B}}^{(s)}$ can be twisted by classes in $H^{n+1}(\mathbb{B}, U(1))$.

- Let G be a finite group. Let $\mathbb{B}$ be the classifying space of G. Then $\mathcal{F}_{\mathbb{B}}^{(s)}$ coincides with Dijkgraaf-Witten TQFT.

  Explicitly calculable. Related to gauge theory.
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- (Conjecture) If $\mathcal{S}$ is a HF simplicial groupoid and $\mathbb{B}$ is the geometric realisation of $\overline{W}(\mathcal{S})$ then $\mathcal{F}_{\mathbb{B}}^{(s)}$ coincides with Porter's homotopy n-type TQFT.

  Also explicitly / combinatorially calculable.

  Conjecture implies all Quinn's TQFTs $\mathcal{F}_{\mathbb{B}}^{(s)}$ are combinatorial.

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- Let M be a manifold.
- A path in M is a piecewise smooth map $\gamma\colon [0,1] \to M$.
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Paths γ_1 and γ_2 are homotopic.

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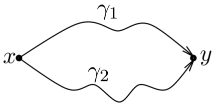

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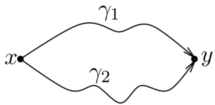

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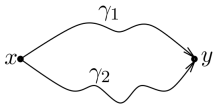

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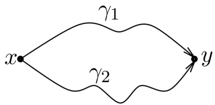

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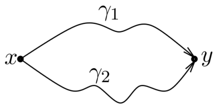

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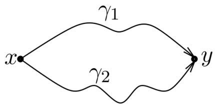

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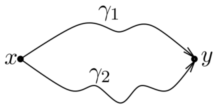

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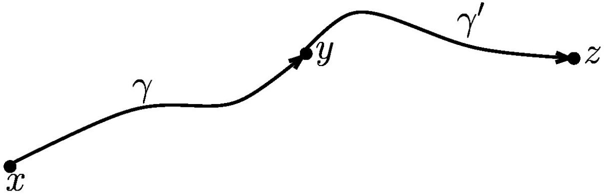

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Let G be a group (G will be finite throughout the talk).
Given a principal G-bundle $P \to M$ – i.e. a gauge field –,
we have the parallel transport (a.k.a. holonomy) of P:

$$\mathcal{F} : \{Paths \quad in \;\; M\} \to G$$
$$\gamma \longmapsto \mathrm{hol}^1(\gamma) = g_\gamma \in G$$

Recall parallel transport preserves concatenation of paths:

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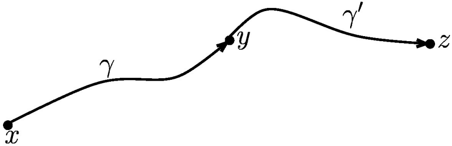

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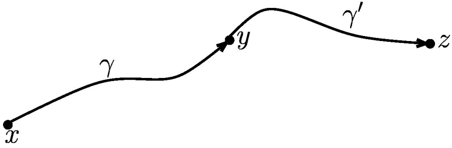

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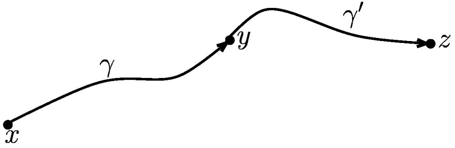

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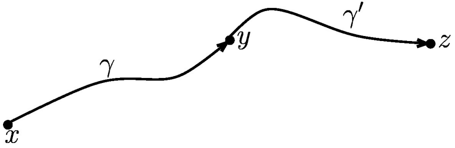

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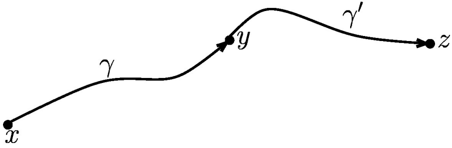

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Conversely, G-connections can be defined from their holonomy.
Since G is finite, and M compact, to reconstruct the G-connection we only need to know the holonomy along a finite number of paths.
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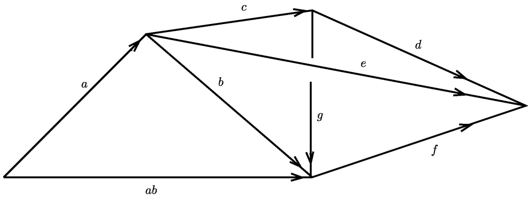

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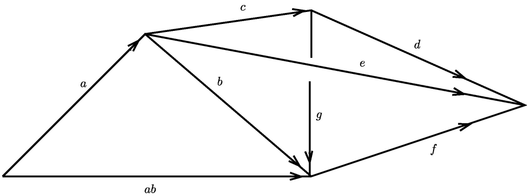

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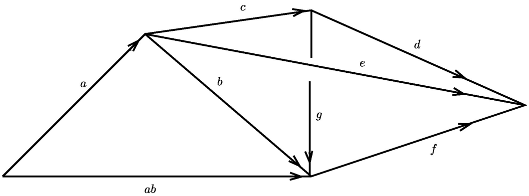

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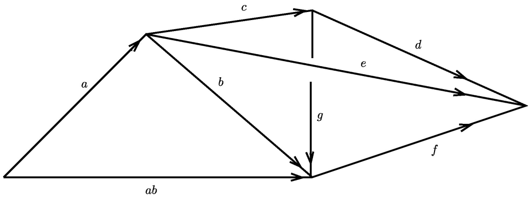

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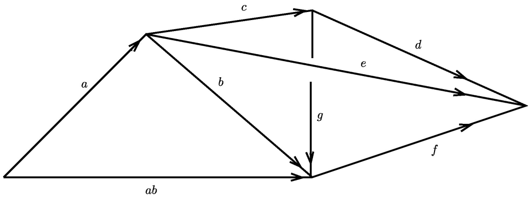

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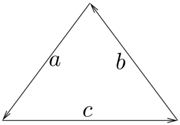

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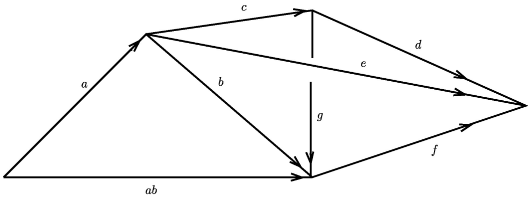

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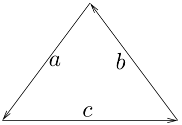

$abc = 1_G$

'Extension' to discrete Higher Gauge Theory

- Higher gauge theory formalises non-abelian holonomy along paths, and also non-abelian holonomy along surfaces.
- Non-abelian holonomy along surfaces is multiplicative with respect to the several ways we can concatenate surfaces.

  (This is why higher category theory arises here.)

- We need a higher order version of a group: called a "2-group".
- 2-groups are equivalent to crossed modules.

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These compose horizontally and vertically:

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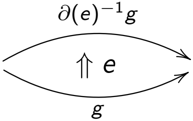

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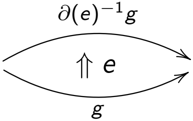

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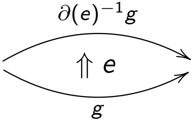

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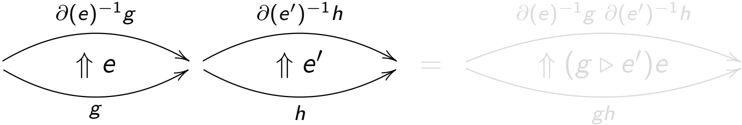

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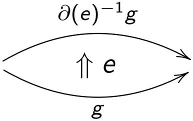

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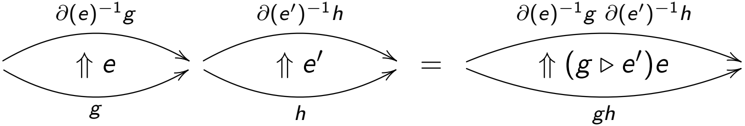

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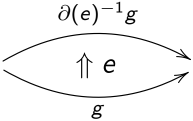
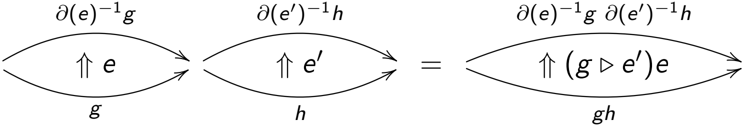
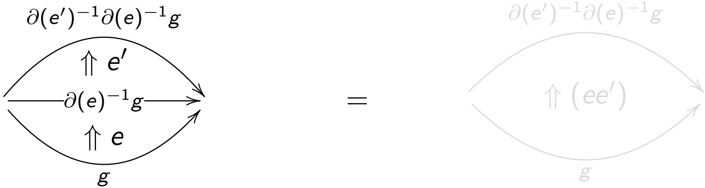


$, \quad g \in G, e \in E.$

These compose horizontally and vertically:





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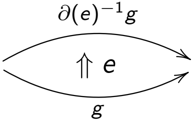

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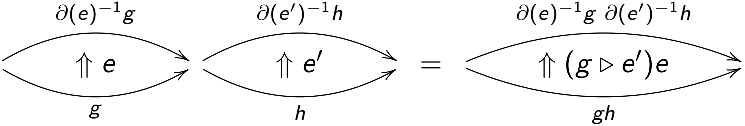

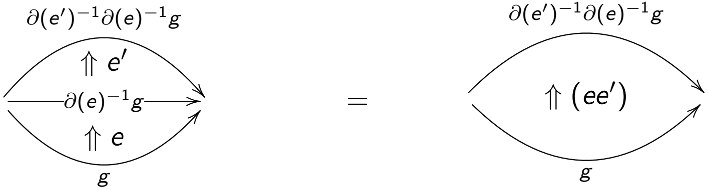

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Horizontal and vertical compositions of bigons in $\mathcal{G}$ are:
associative, and have units and inverses.
The interchange law is satisfied. This means that the evaluation of

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As a consequence, evaluations of more complicated diagrams like:

do not depend on the order whereby we apply compositions.
This leads to a notion of non-abelian multiplication along surfaces.
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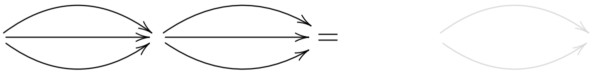

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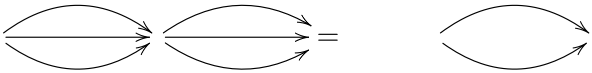

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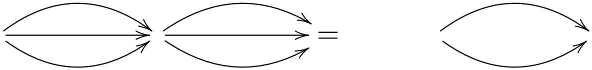

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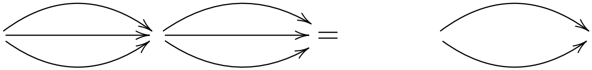

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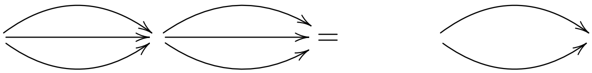

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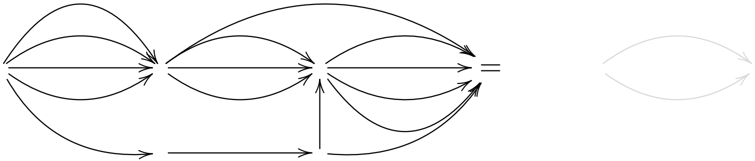

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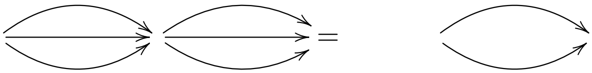

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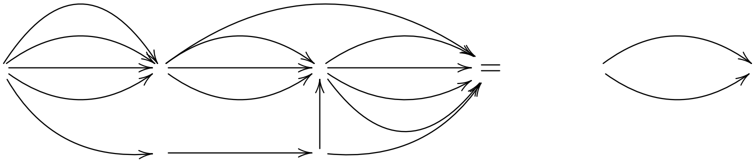

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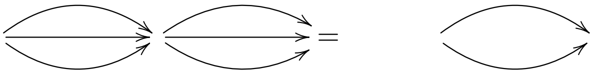

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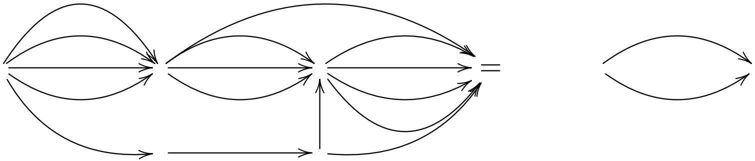

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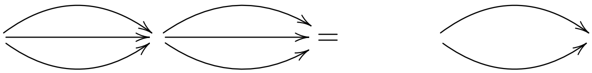

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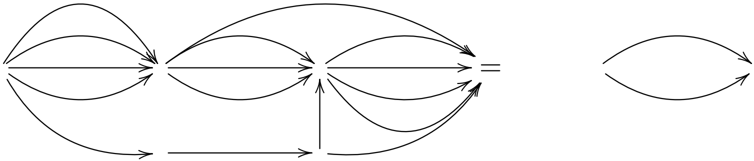

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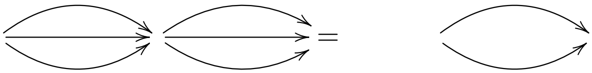

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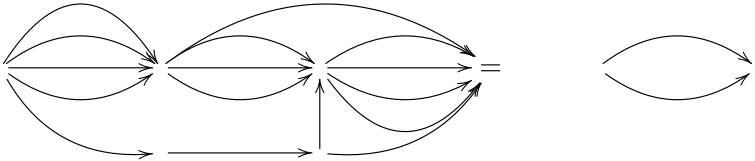

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2-dimensional holonomy

A geometric bigon on in a manifold M is given by:
Two paths $\gamma, \gamma' \colon [0,1] \to M$, with the same initial and end-point.
A homotopy (i.e. a 'surface') $\Sigma \colon [0,1]^2 \to M$, connecting γ and γ'.
Σ is considered up to homotopy relative to $\partial([0,1]^2)$.
Geometric bigons are represented as:

$$x \underset{\gamma}{\overset{\gamma'}{\underset{\Uparrow \Sigma}{\rightrightarrows}}} y,$$

Geometric bigons can be concatenated horizontally and vertically.

- **Definition** Let M be a manifold; $\mathcal{G}$ a crossed module.
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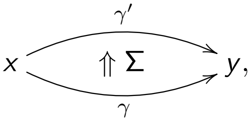

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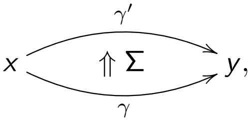

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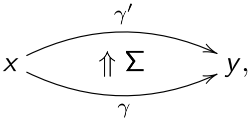

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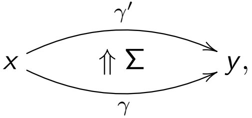

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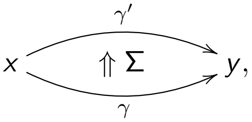

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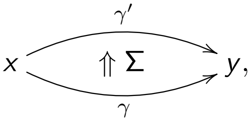

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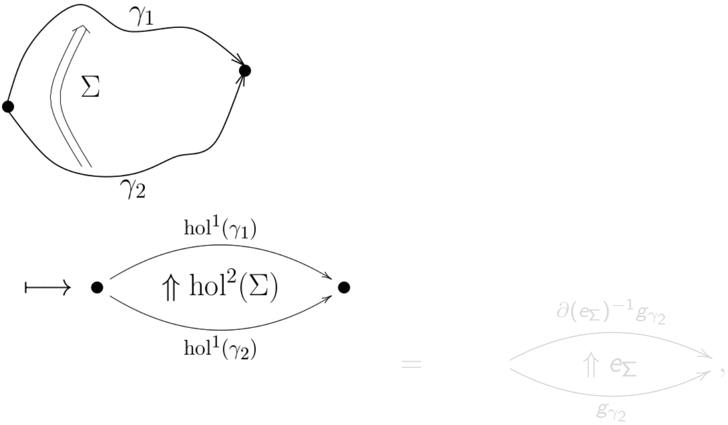

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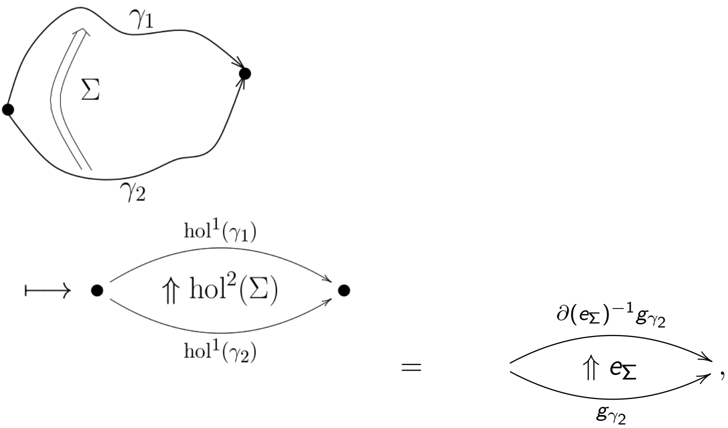

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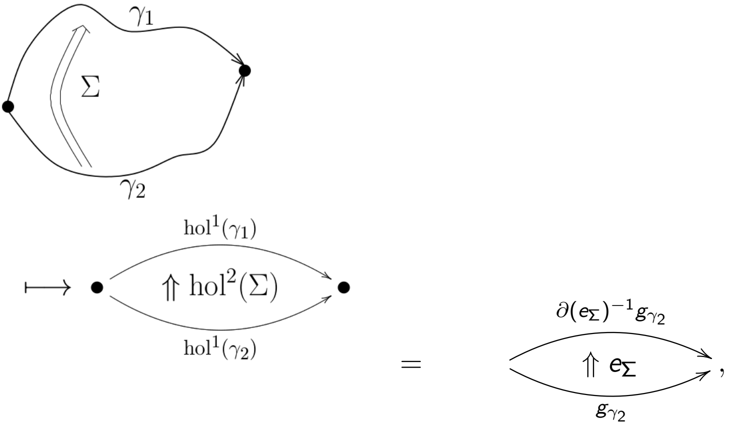

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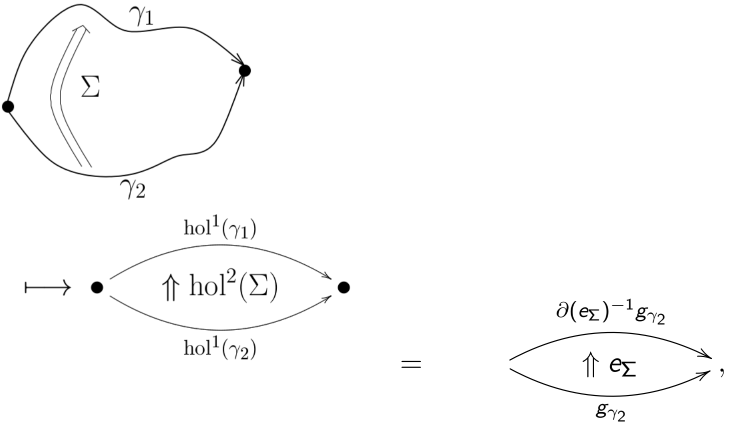

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Let M be a compact manifold, possibly with boundary.

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If we have a configuration like:

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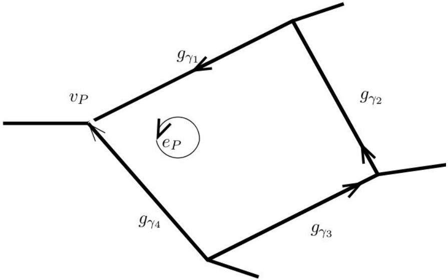

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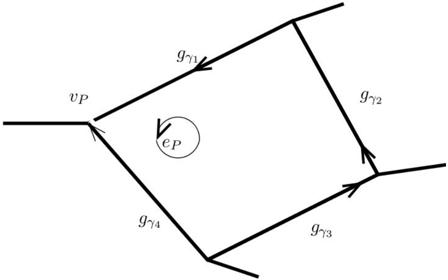

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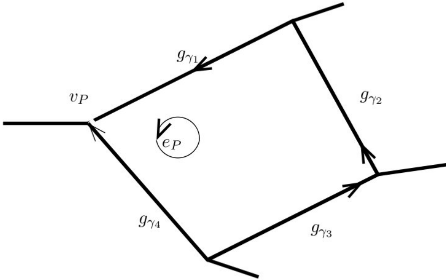

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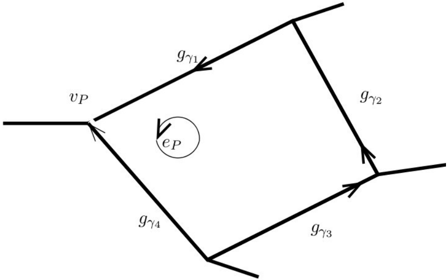

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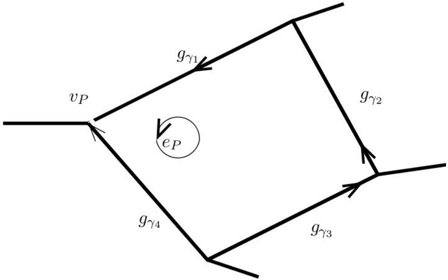

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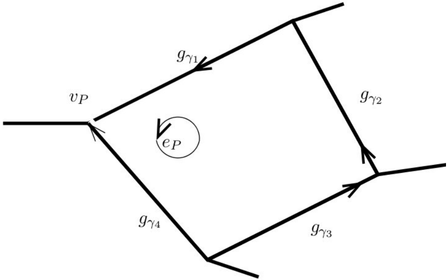

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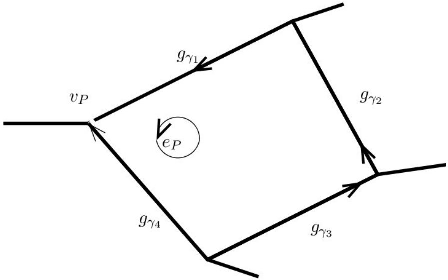

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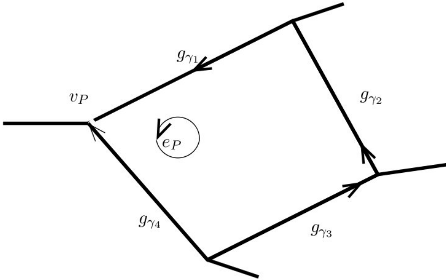

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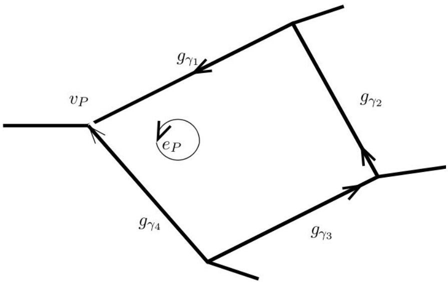

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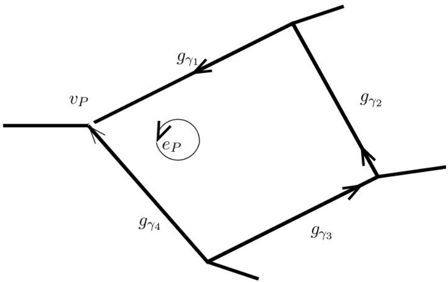

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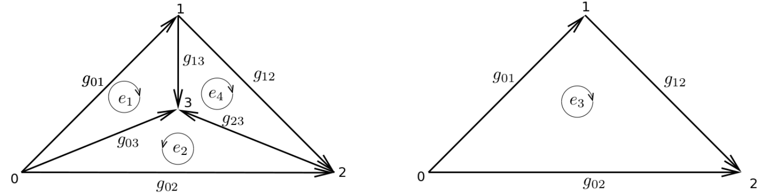

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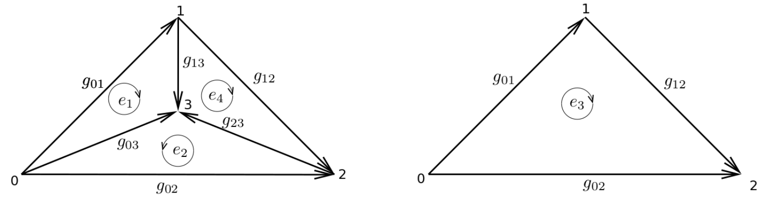

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Crossed complexes

A crossed complex is given by a complex

$$\mathcal{C} := \ldots \xrightarrow{\partial} C_n \xrightarrow{\partial} C_{n-1} \xrightarrow{\partial} \ldots \ldots \xrightarrow{\partial} C_2 \xrightarrow{\partial} C_1$$

of groupoids, all with object set C_0. Such that:

- All groupoids for C_i, $i \geq 2$ are totally disconnected.
- All boundary maps are the identity over the object C_0.
- We have an action of C_1 over on all groupoids C_i, $i \geq 2$
- All boundary maps preserve the action.
- Peiffer 1: If $x \xrightarrow{g} y \in C_1$ and $K \in C(y, y)$ then:

$$\partial(g \triangleright K) = g\partial(K)g^{-1}$$

- Peiffer 2: If $K, L \in \mathbb{C}_2(y, y)$ then $\partial(K) \triangleright L = KLK^{-1}$
- The action of $\partial(C_2)$ is trivial on all groupoids C_i for $i \geq 3$.
- C_i is abelian if $i \geq 3$.

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The category of crossed complexes is equivalent to the category of strict ω-groupoids.

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Fundamental crossed complexes and nerves

Theorem (Brown-Higgins)

Let X be a CW-complex. Then the sequence

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is a totally free crossed complex with object set X_0.

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is the simplicial sets given by all maps $\Pi(\Delta(n)) \to \mathcal{C}$.

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The homotopy groups of the realisation of $\mathcal{NC}$ coincide with the homology groups of $\mathcal{C}$.

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Let M be a manifold with triangulation t.

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- a map f_0: Vertices$(M_t) \to C_0$
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$$f_0(v_0) \xrightarrow{f_1(\gamma)} f_0(v_1) \text{ at each edge } v_0 \xrightarrow{\gamma} v_1. \text{ of } M_t$$

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$$\begin{array}{ccc} f_0(v_0) & \xrightarrow{f_1(\gamma_{01})} & f_0(v_1) \\ & f_2(\Delta_{012}) & \\ {\scriptstyle \partial(f_2(\Delta_{123}))f_1(\gamma_{02})} \searrow & & \swarrow {\scriptstyle f_1(\gamma_{12})} \\ & f_0(v_2) & \end{array} \quad \text{at each triangle} \quad \begin{array}{ccc} v_0 & \xrightarrow{\gamma_{01}} & v_1 \\ & \Delta_{012} & \\ {\scriptstyle \gamma_{02}} \searrow & & \swarrow {\scriptstyle \gamma_{12}} \\ & v_2 & \end{array}$$

Rule: boundary of element associated to a $n+1$-simplex is the n dimensional holonomy around boundary of simplex.

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Rule: boundary of element associated to a $n+1$-simplex is the n dimensional holonomy around boundary of simplex.

$\mathcal{C}$-colourings ($\mathcal{C}$ a crossed complex)

Let M be a manifold with triangulation t.
Let M_t be corresponding CW-complex. Consider:

$$\mathcal{C} = \ldots \xrightarrow{\partial} C_n \xrightarrow{\partial} C_{n-1} \xrightarrow{\partial} \ldots\ldots \xrightarrow{\partial} C_2 \xrightarrow{\partial} C_1$$

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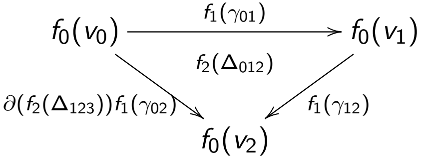
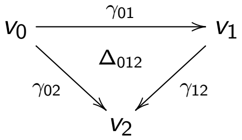


at each triangle



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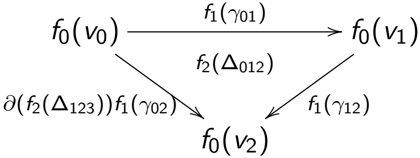

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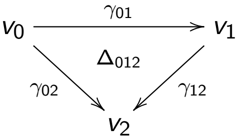

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Calculation of Quinn $\mathcal{F}^{(s)}_{\mathbb{B}}$ for $\mathbb{B} = |N(\mathcal{C})|$.

Let $\mathcal{C}$ be a pointed homotopically finite crossed complex
Hence $\mathbb{B} := |\mathcal{N}\mathcal{C}|$ is a homotopically finite space.

Theorem (Martins/Porter (following Brown-Higgins))

Let A be a compact n-manifold with a triangulation t. Then

$$\mathcal{F}^{(s)}_{\mathbb{B}}(A) = \mathbb{C}(\pi_0(\mathrm{CRS}(\Pi(A_t), \mathcal{C})).$$

Here $\mathrm{CRS}(\_, \_)$ is internal-hom in the cat. of crossed complexes.

In particular a basis of $\mathcal{F}^{(s)}_{\mathbb{B}}(A)$ consists of equivalence classes of $\mathcal{C}$-colourings of A_t up to 'gauge transformations' of all orders.

Note $\mathrm{CRS}(\Pi(A_t), \mathcal{C})$ is the crossed complexes of all maps $f : \Pi(A_t) \to \mathcal{C}$ and their homotopies / natural transformations of all orders.
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Given $f \colon \Pi(A_t) \to \mathcal{C}$ and $f' \colon \Pi(A_t) \to \mathcal{C}$

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